

Fermions can be more nonlocal than qubits

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Based on arXiv:2606.12363 “Fermions are fundamentally more nonlocal than Bosons”

with Fatemeh Moradi Kalarde, Sadra Boreiri, Lucas Tendick, Salman Beigi, Paolo Perinotti, Tommaso Guaita, and Marc-Olivier Renou

Our result: Bell-like theorem within quantum theory

Classical physics

Quantum physics

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**Classical particles as
info carriers – bits**

Quantum physics

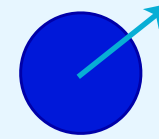
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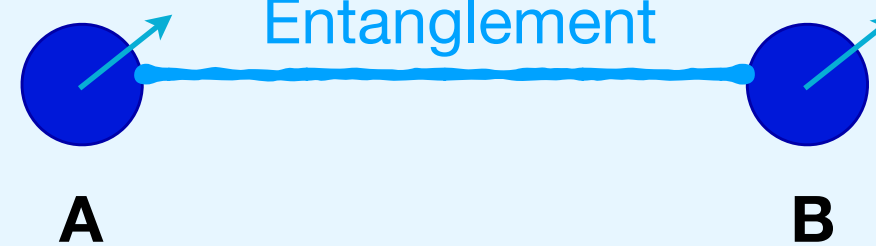


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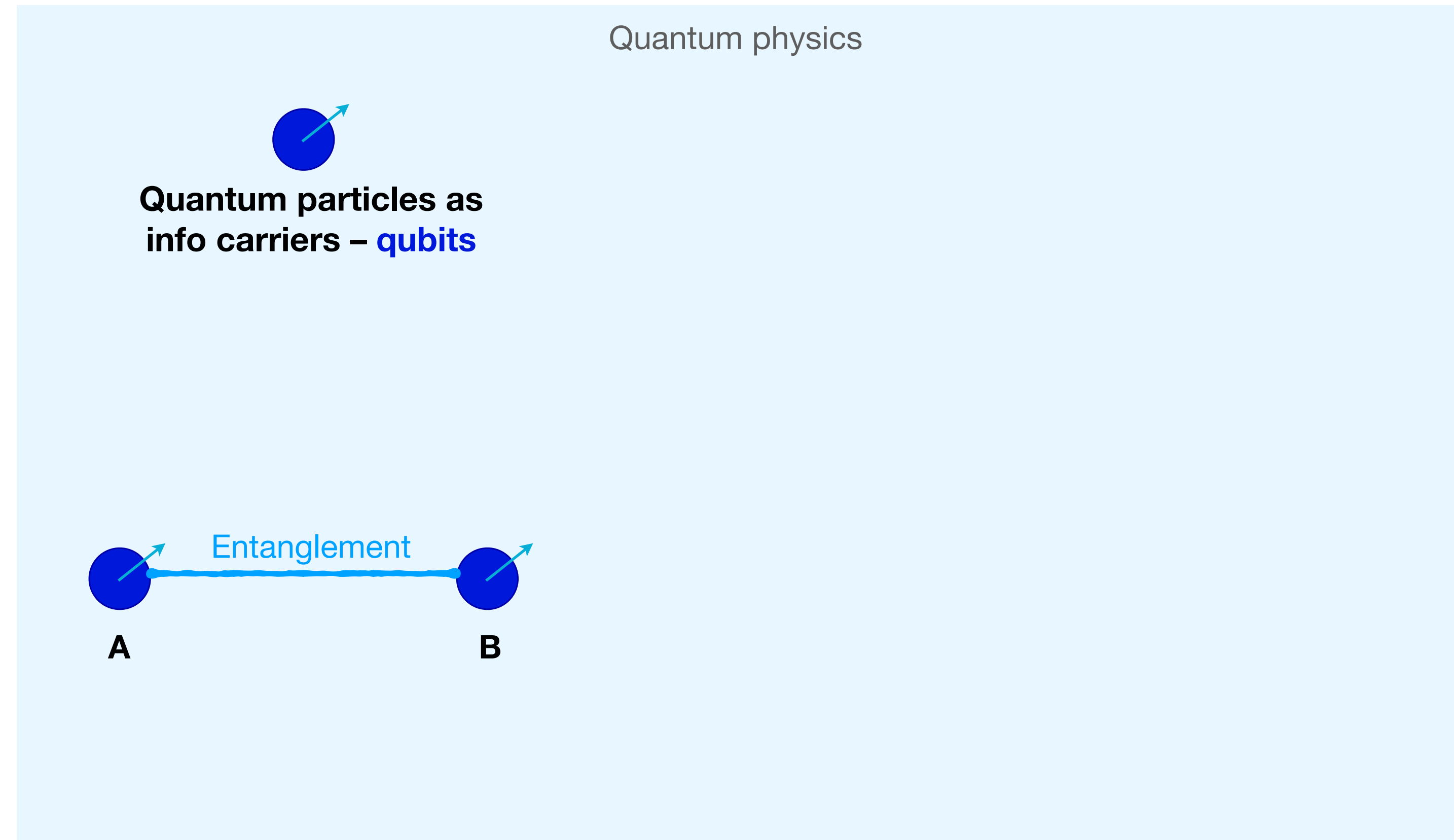
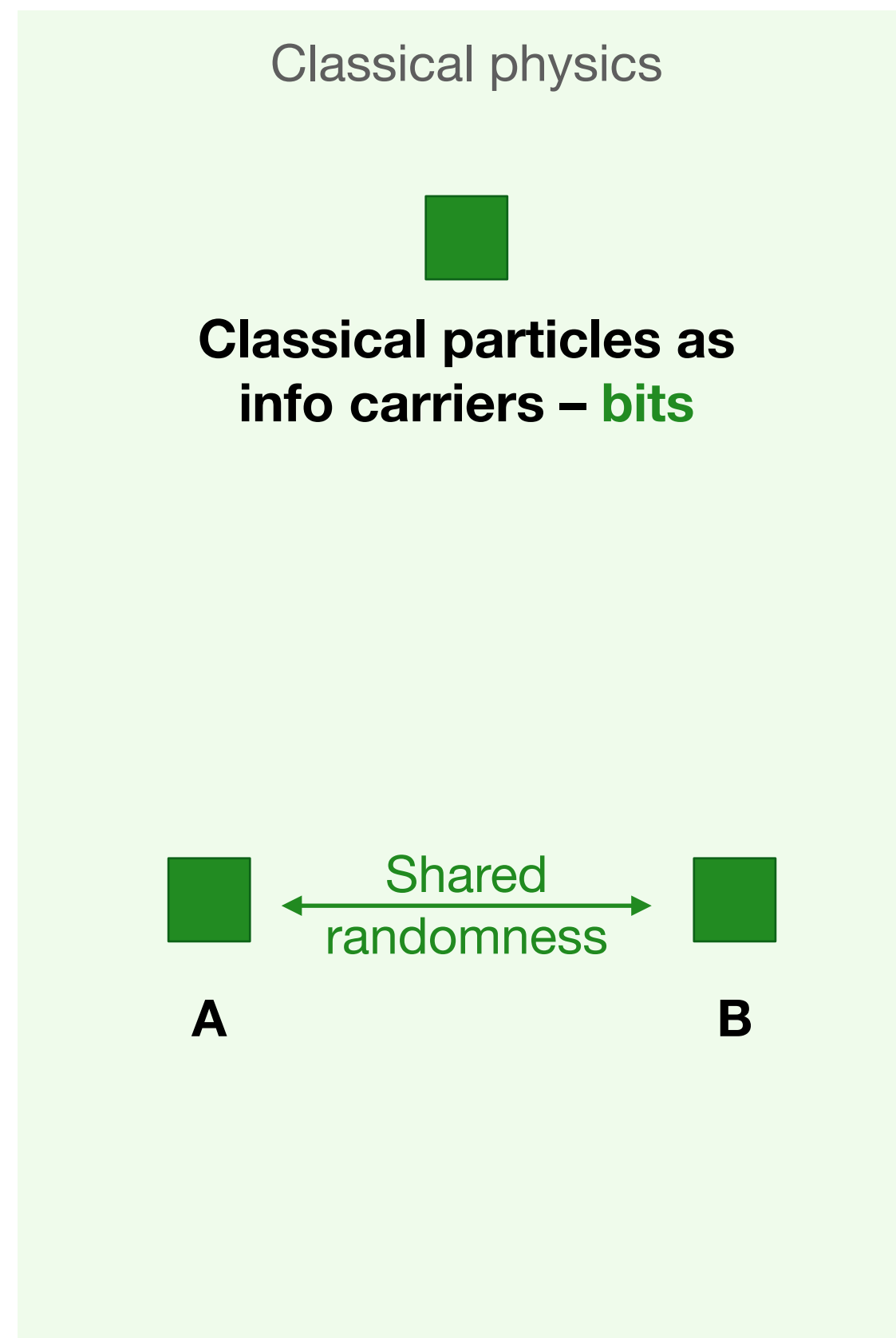


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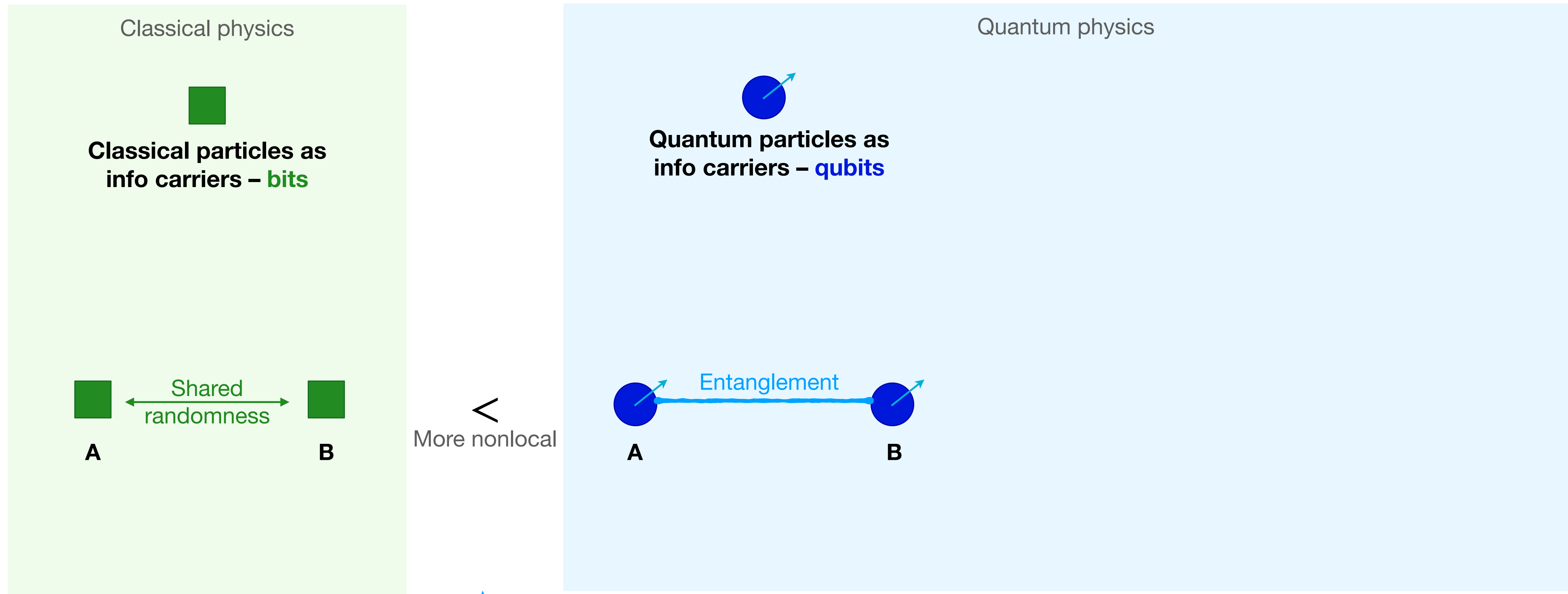
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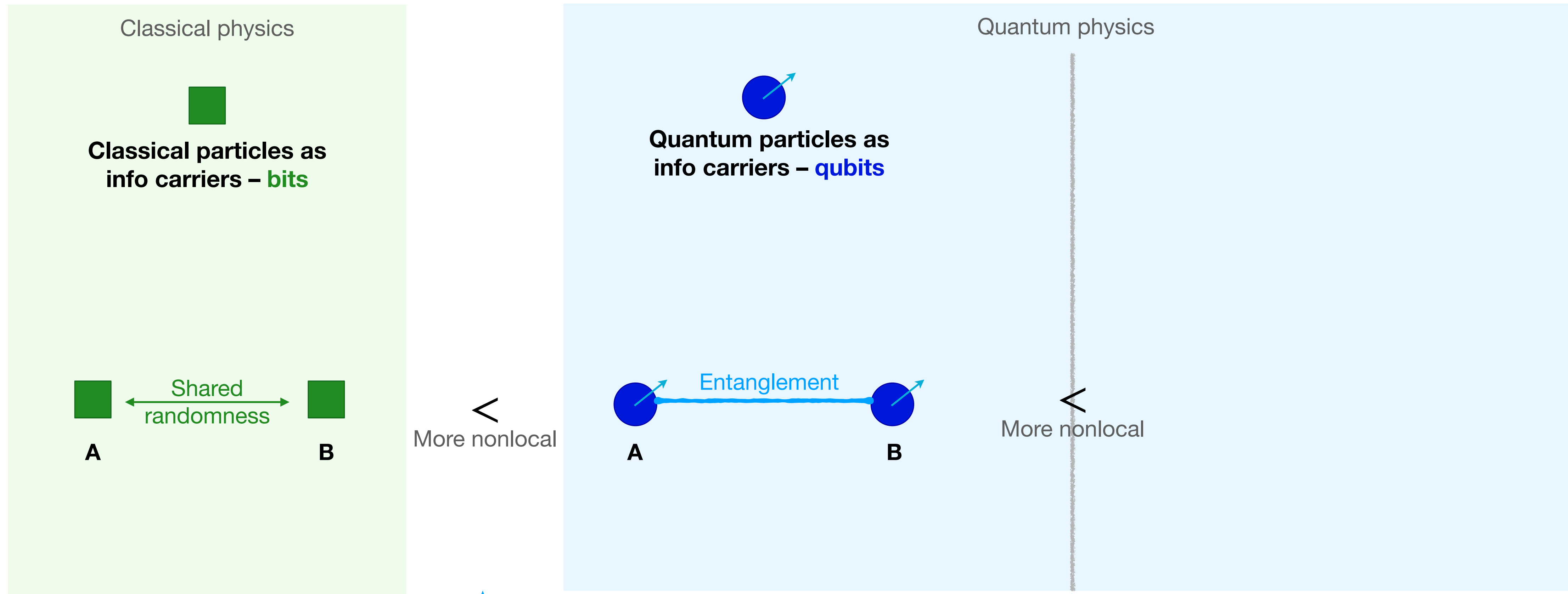
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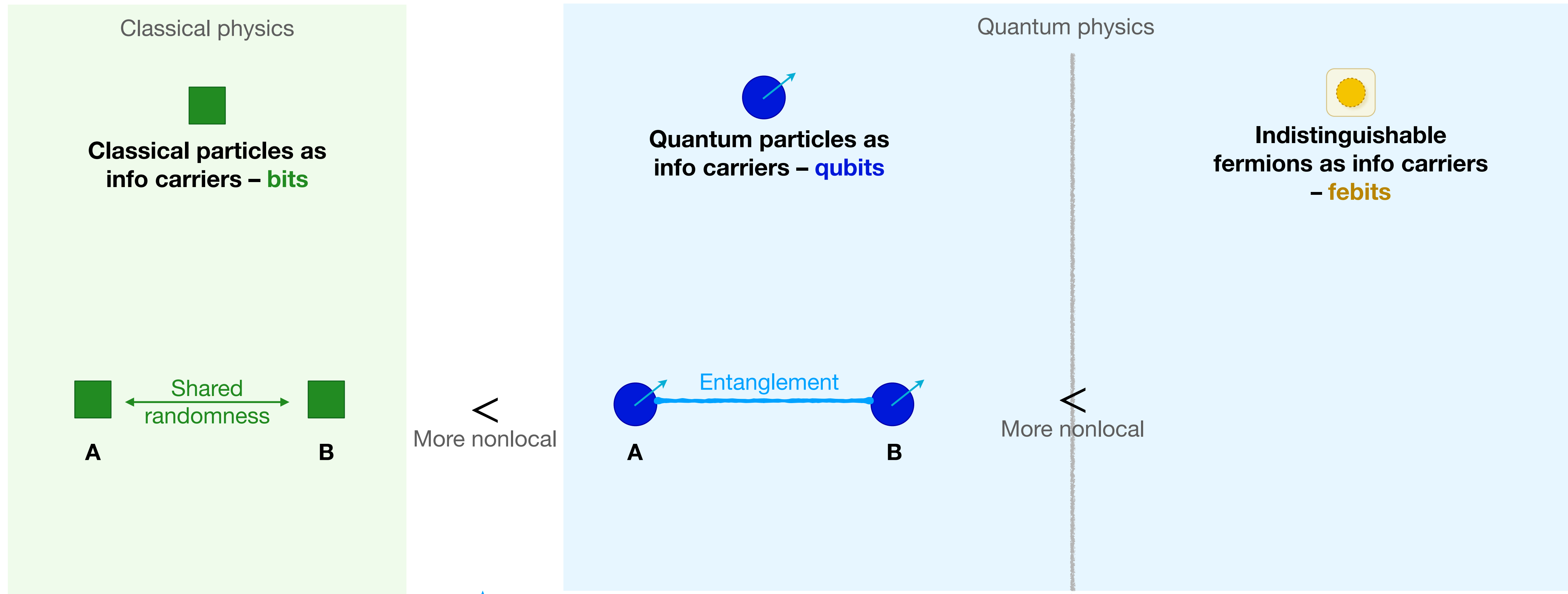
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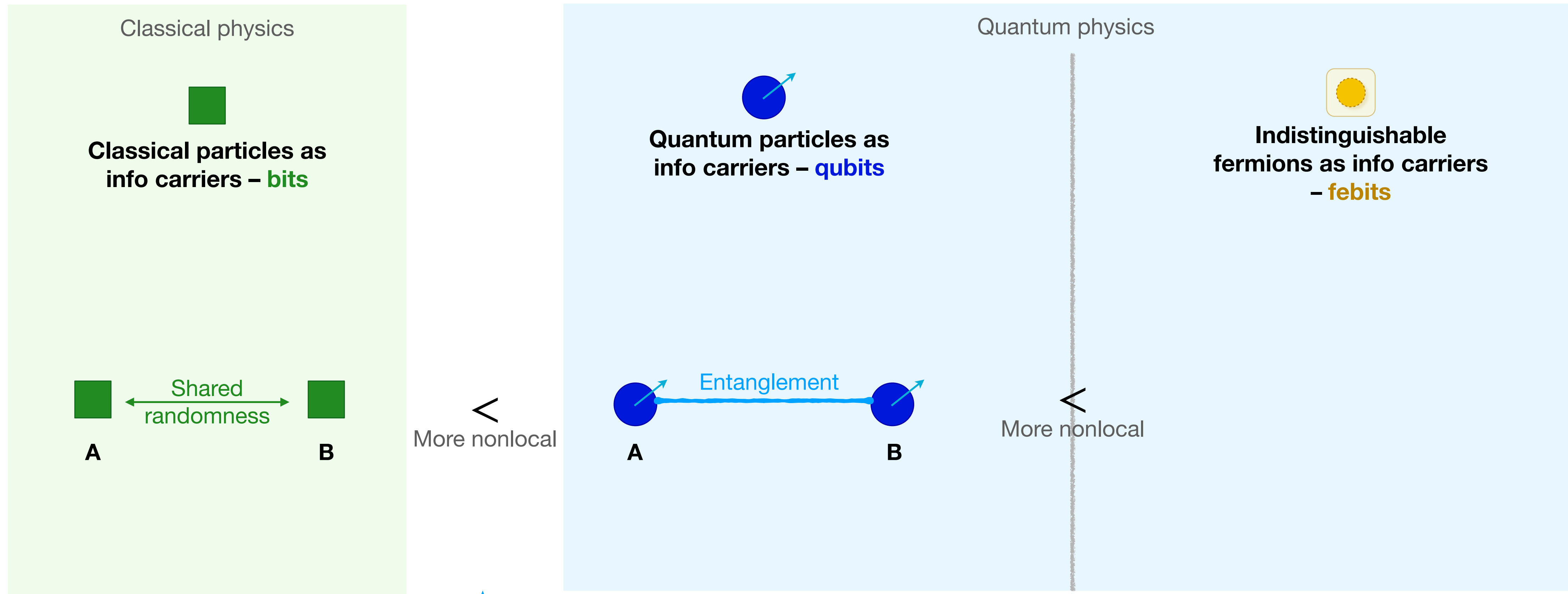
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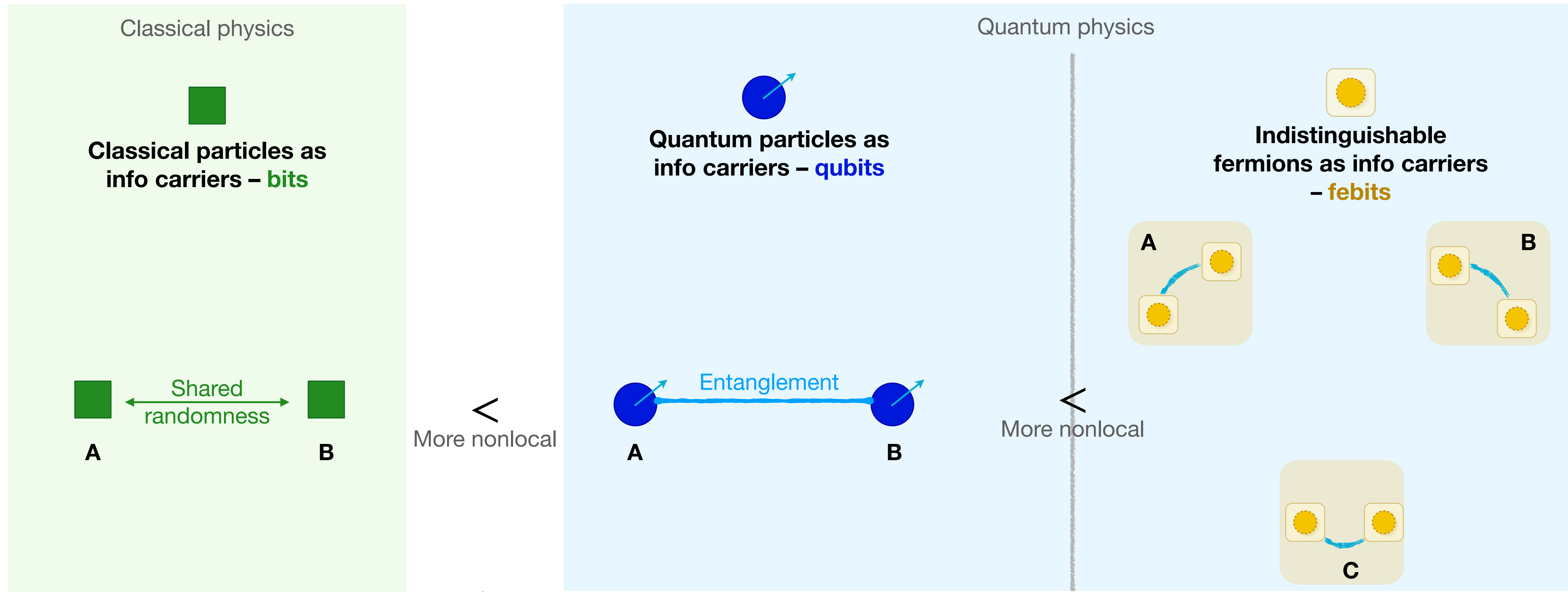
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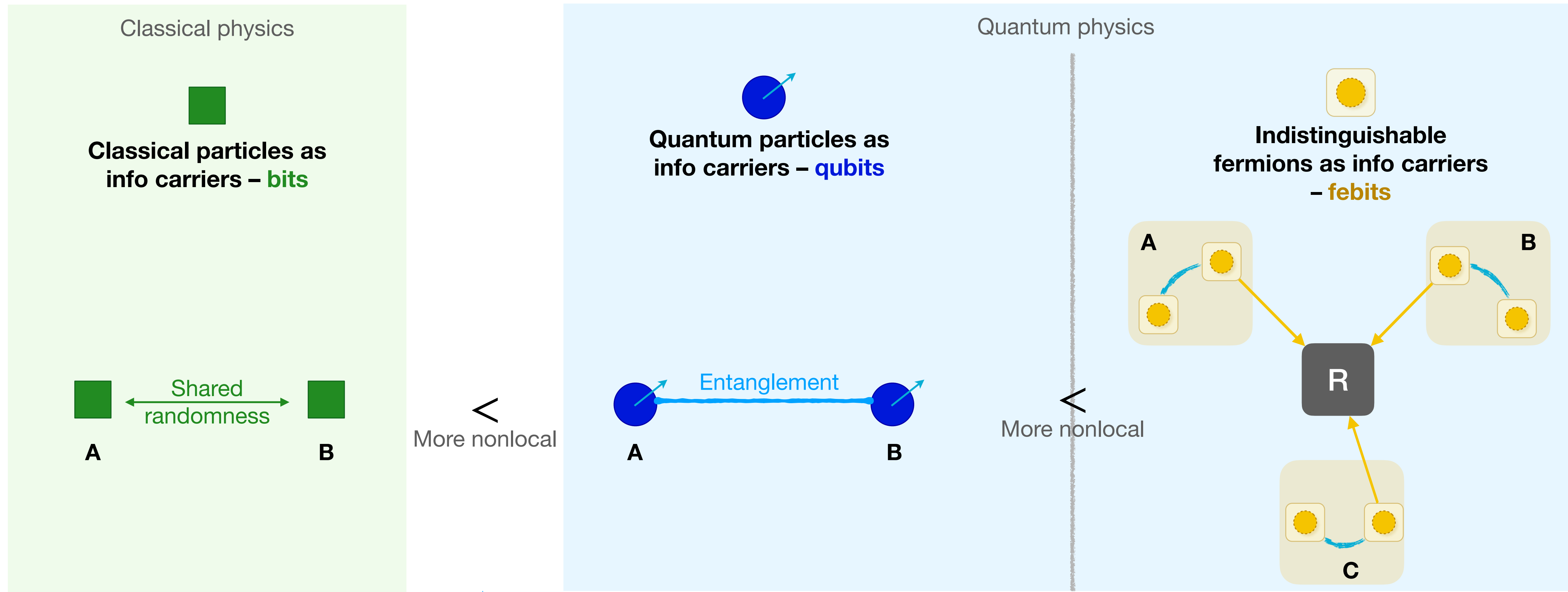
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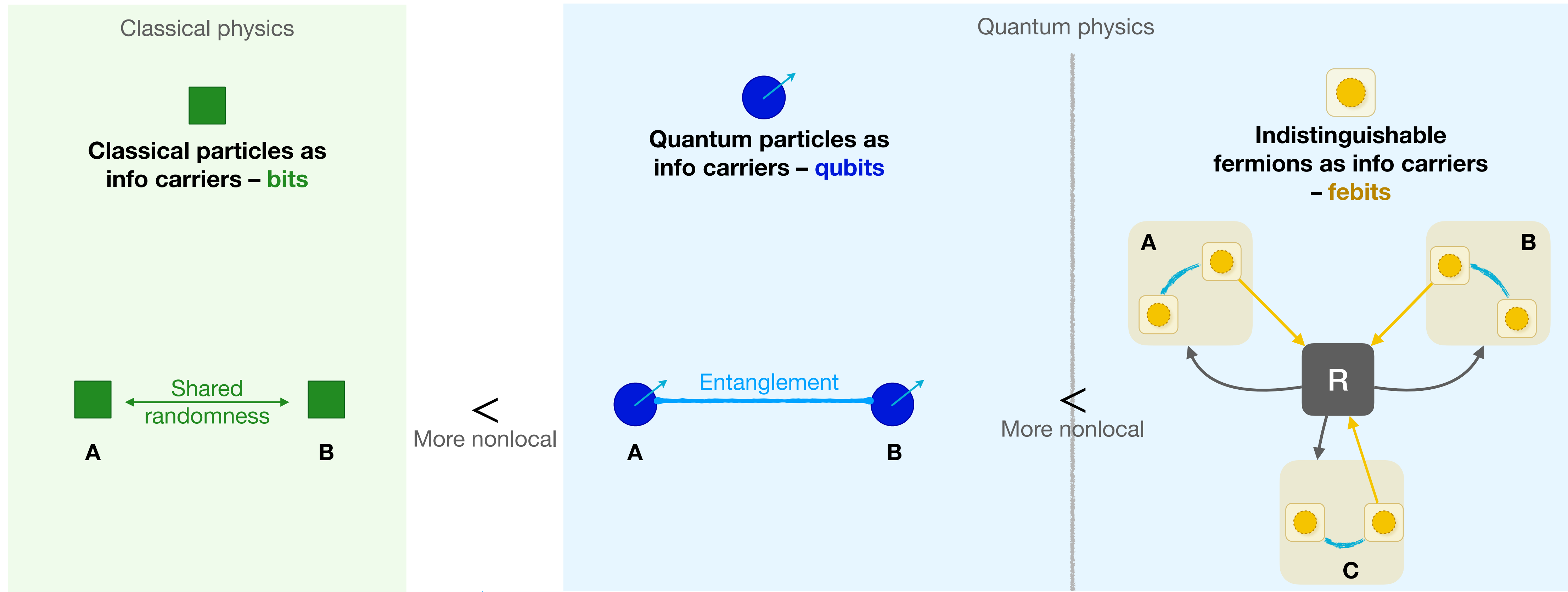
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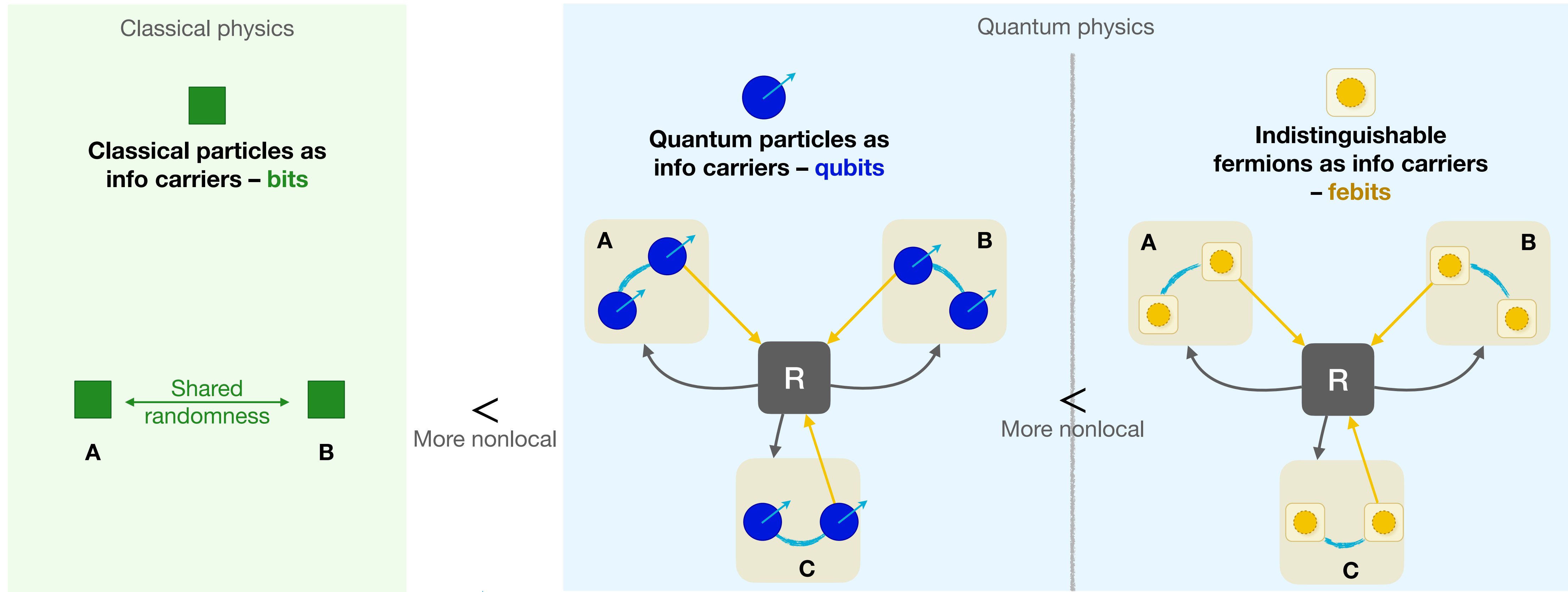
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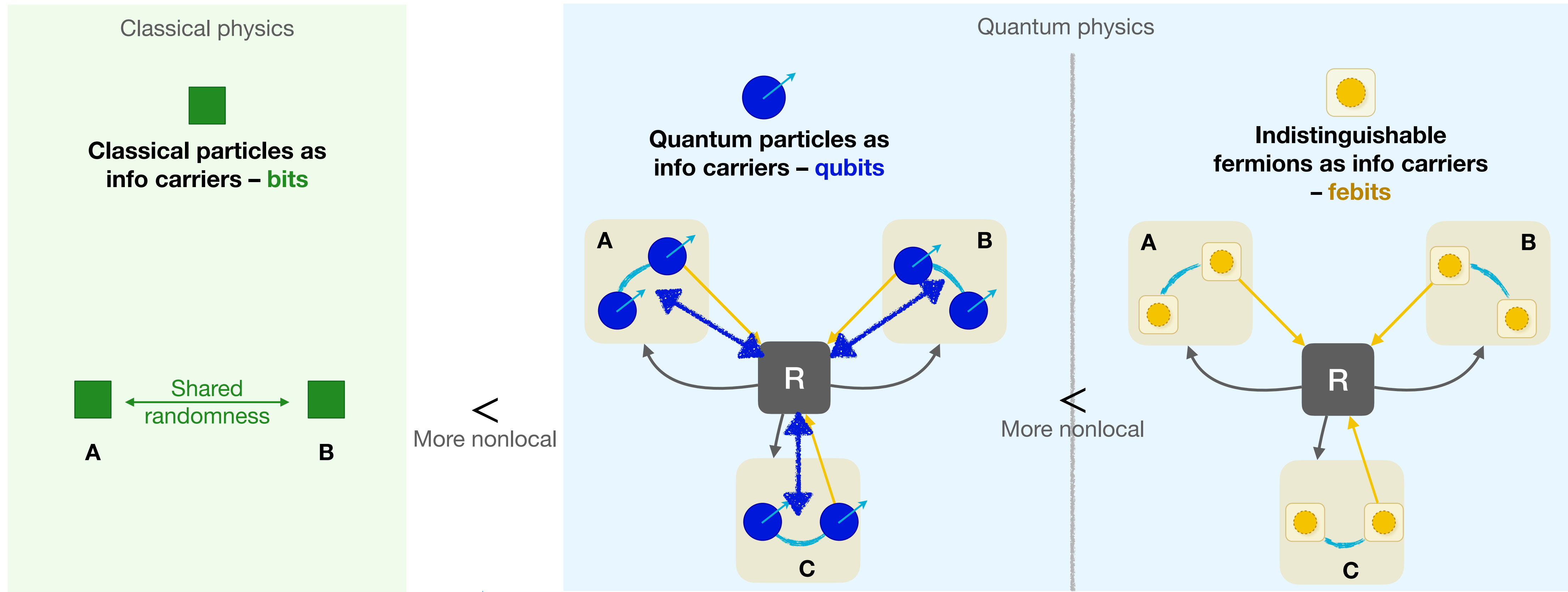
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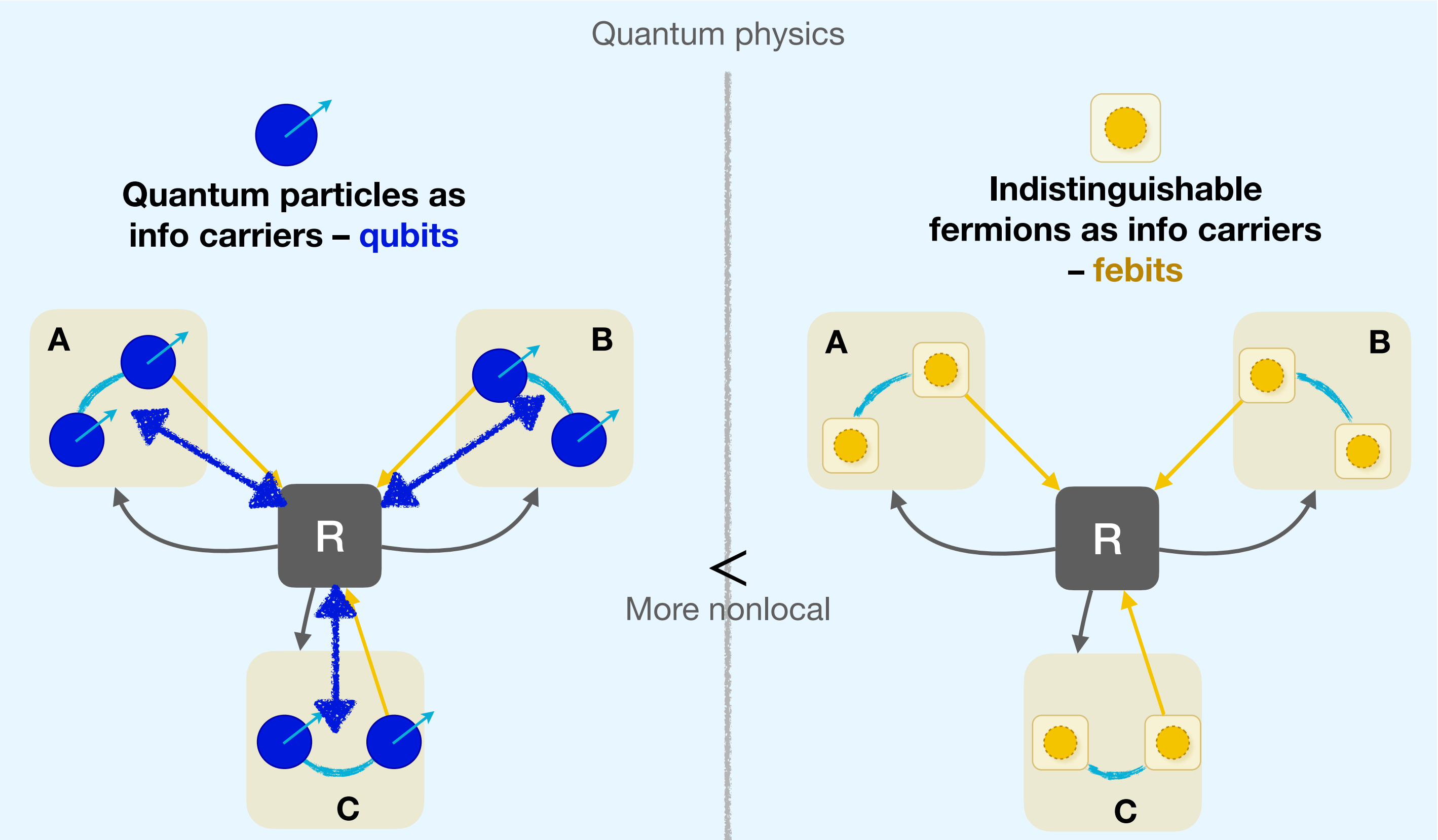
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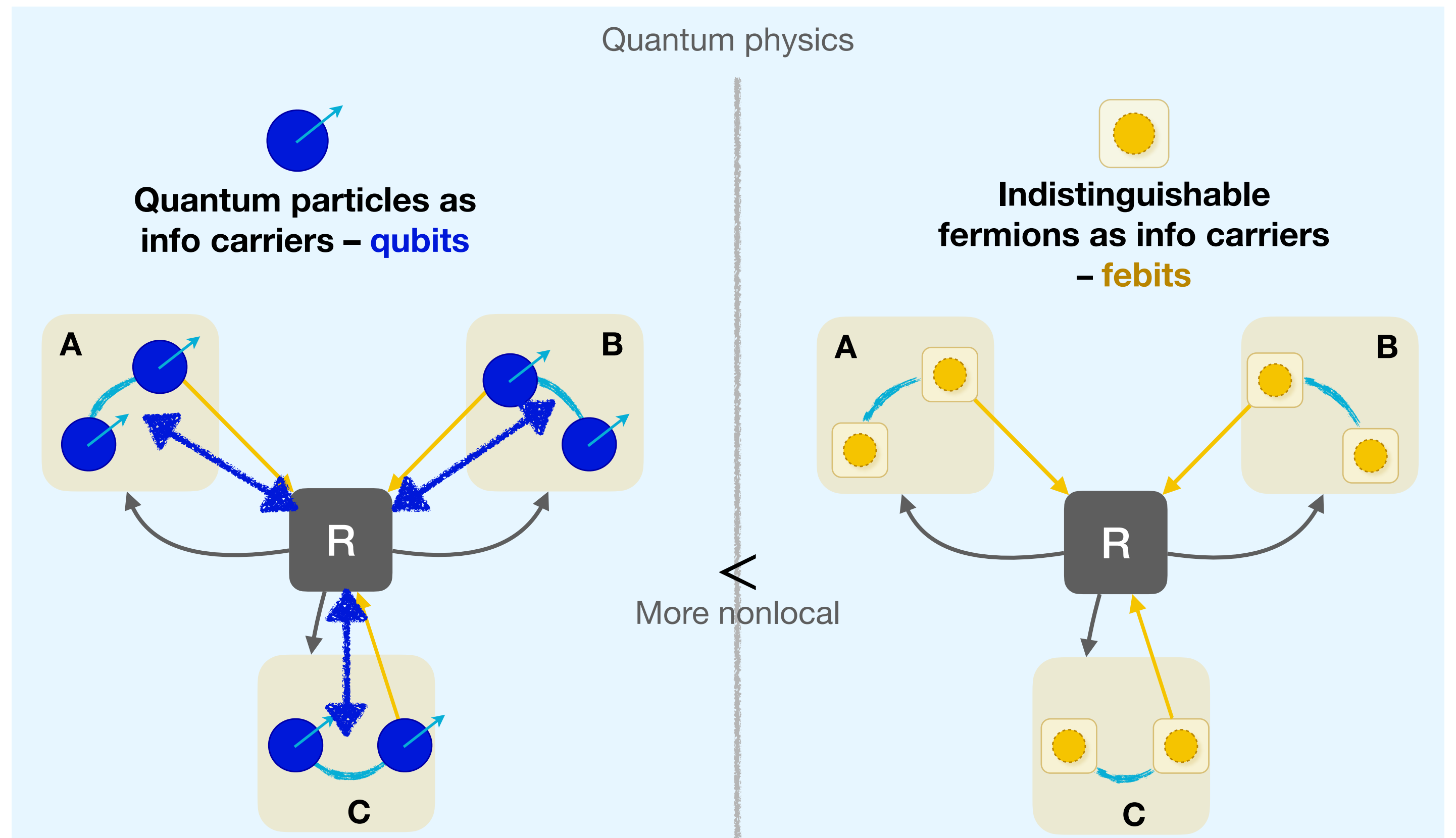
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Outline



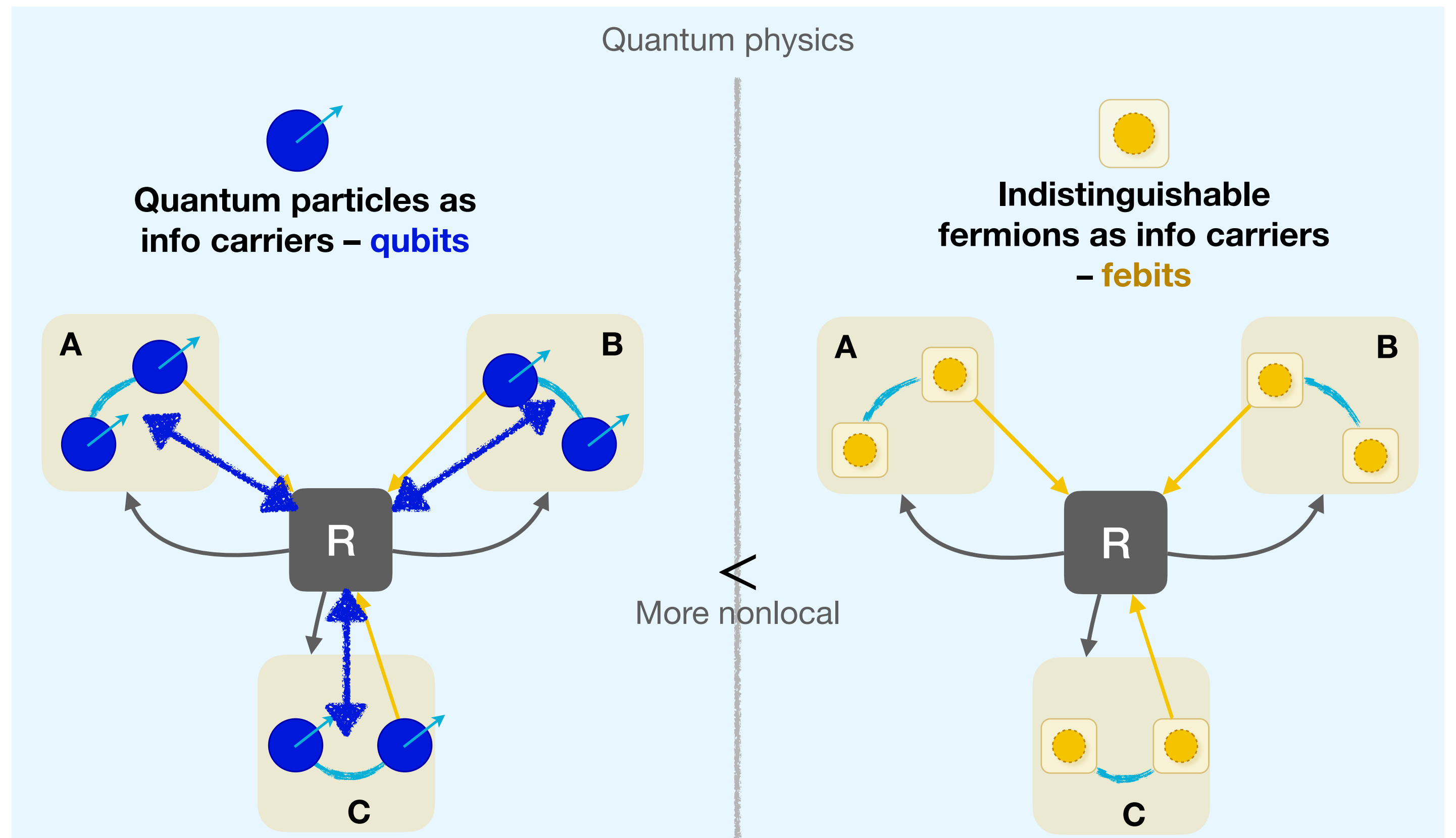
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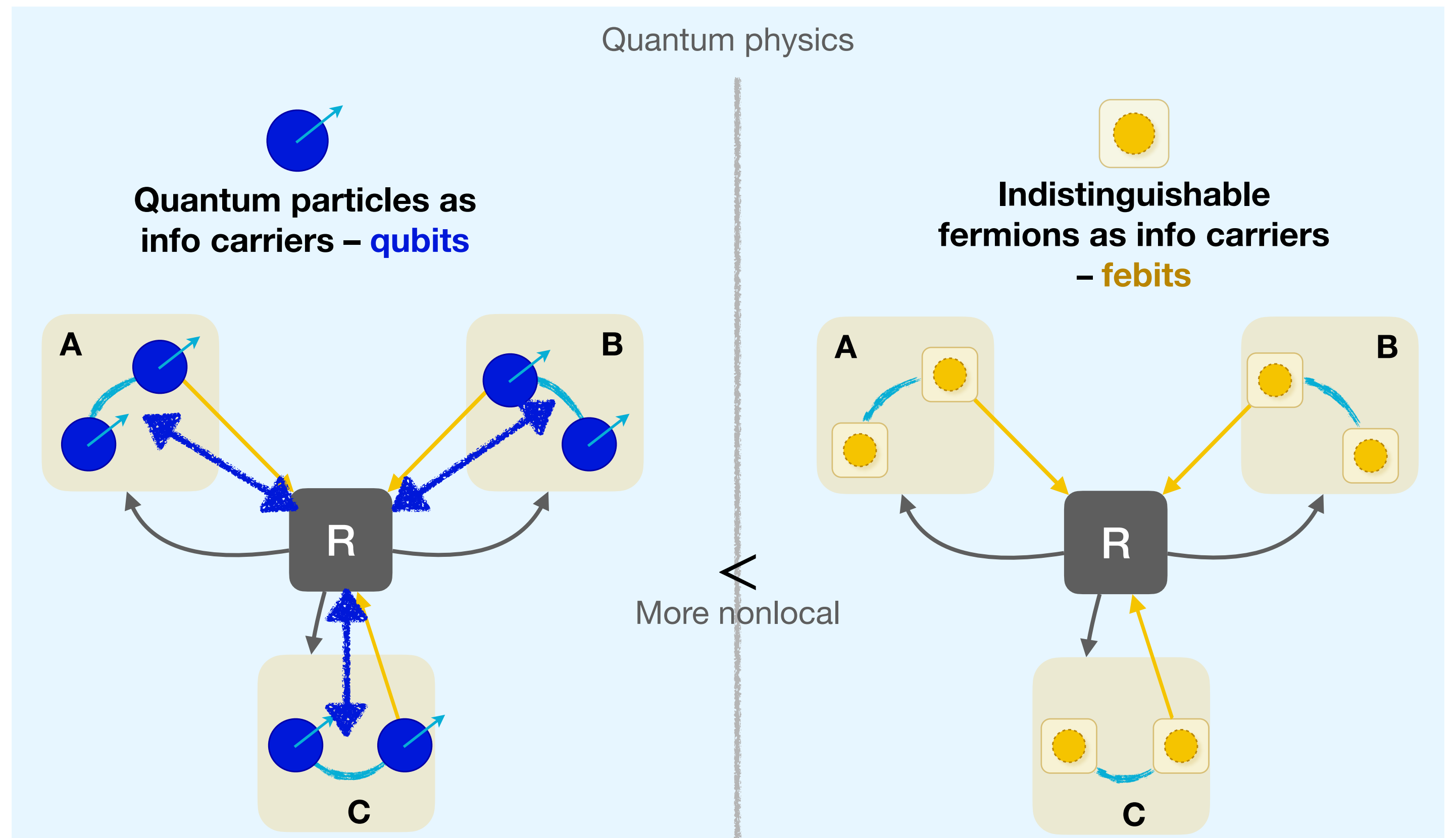
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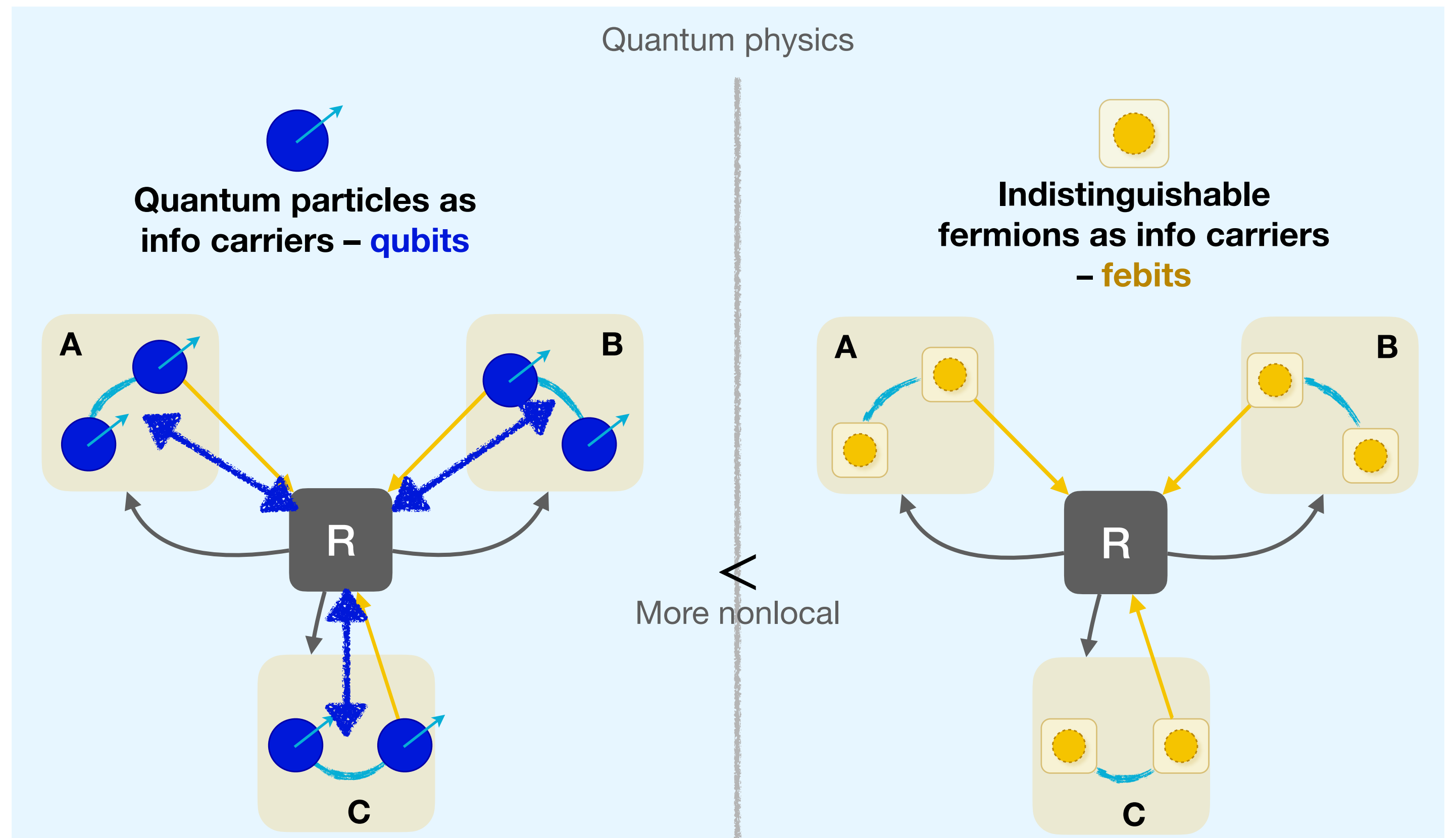
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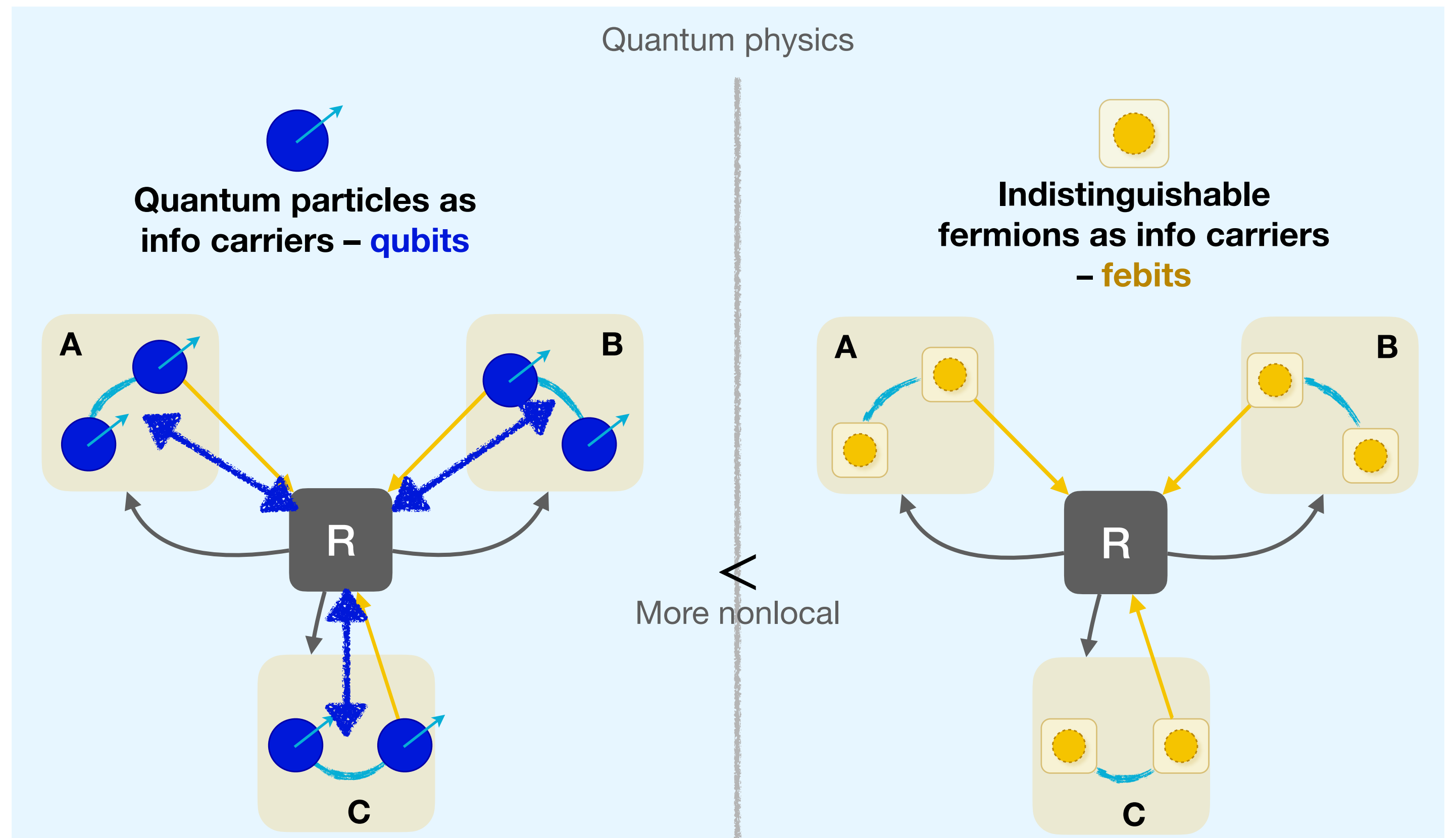
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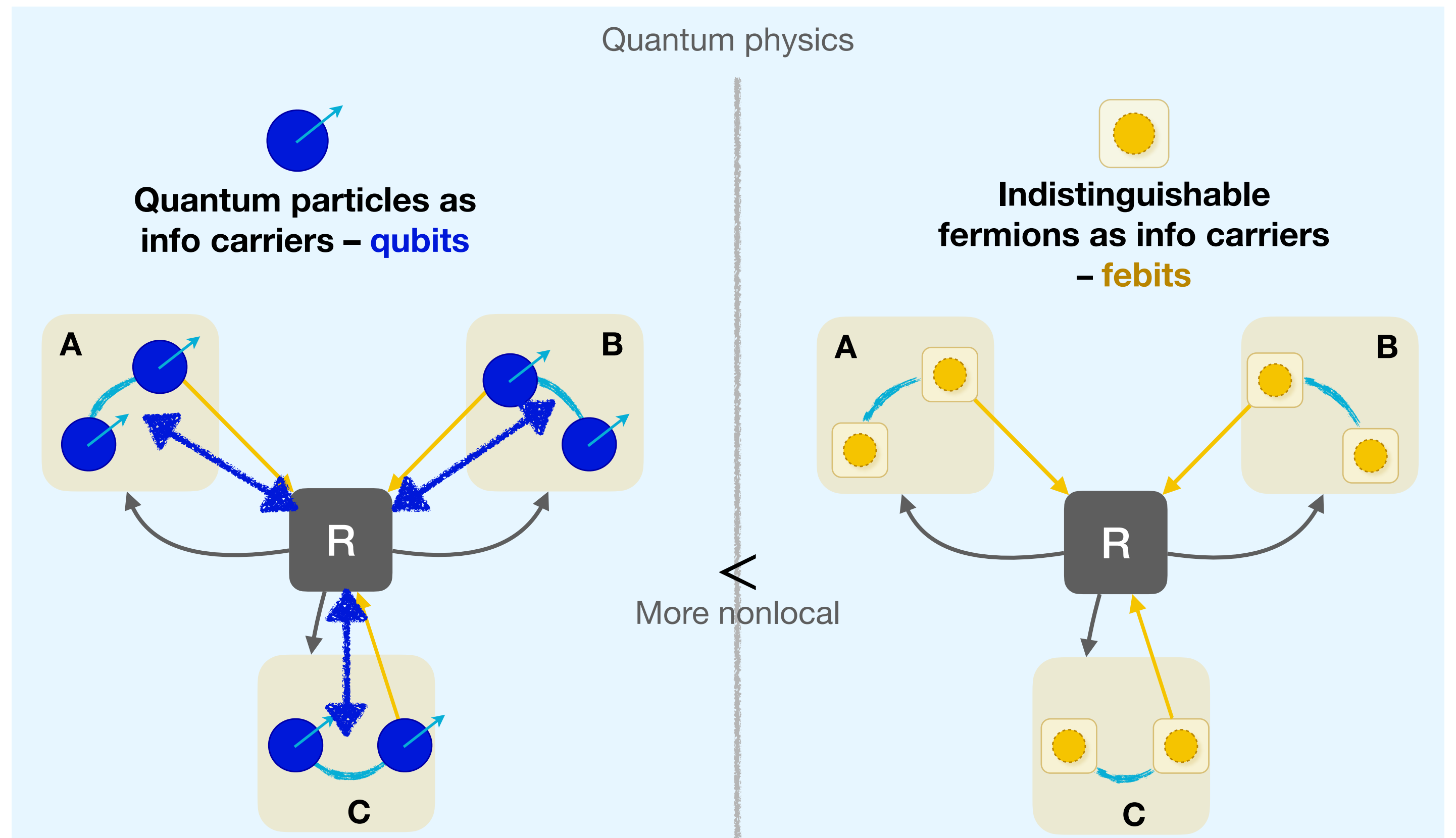
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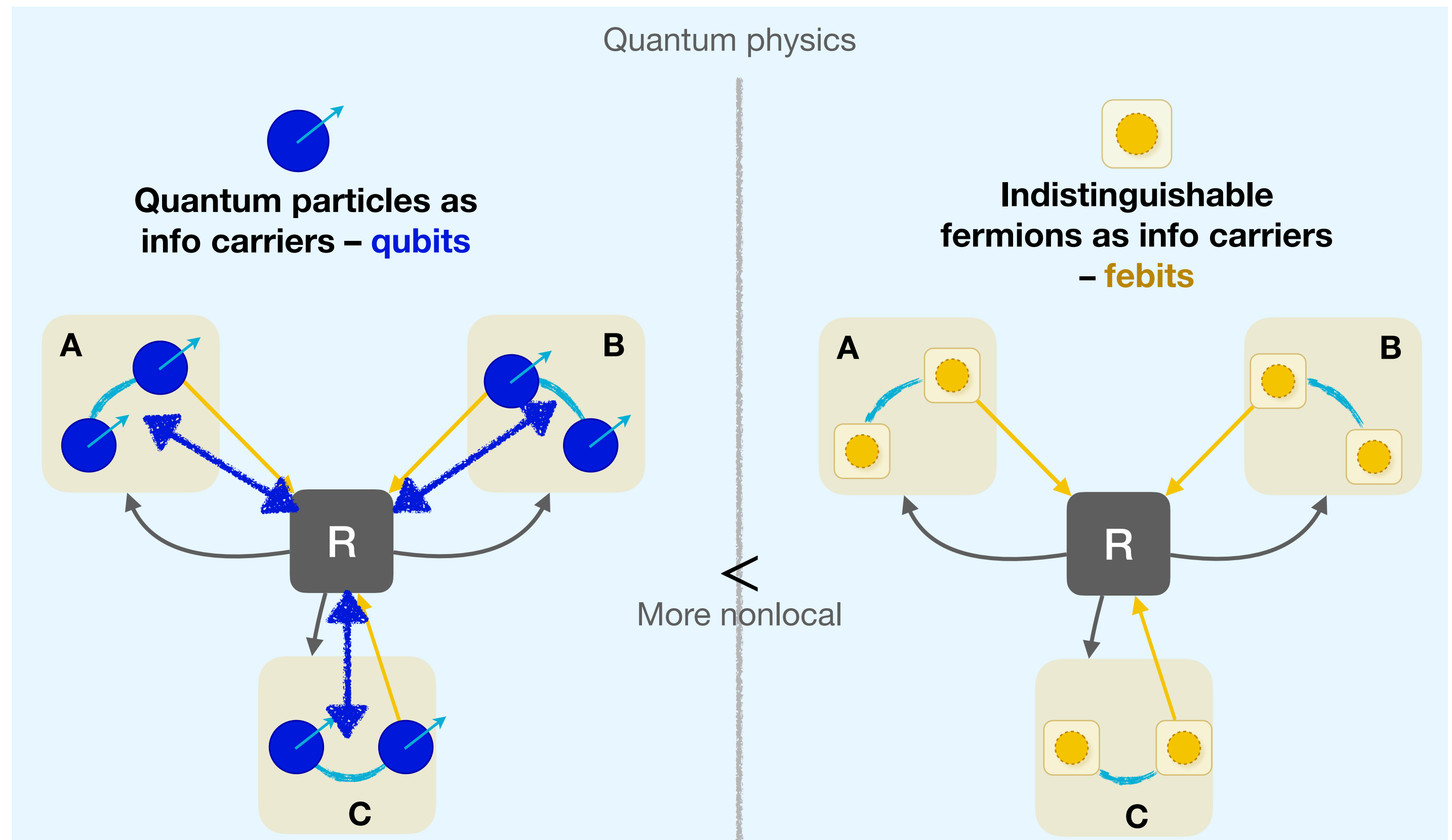
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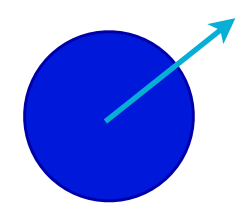


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- Conclusions



Qubits vs febits

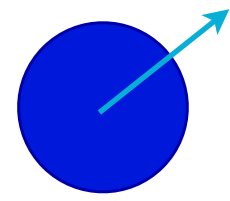


qubits



febits

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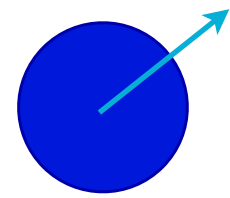
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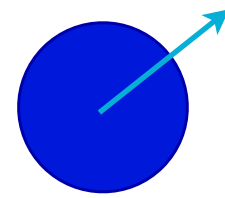
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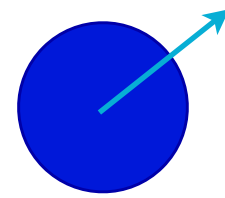
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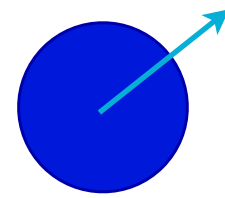
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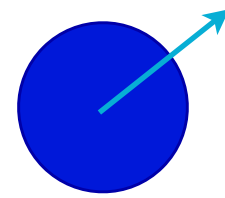
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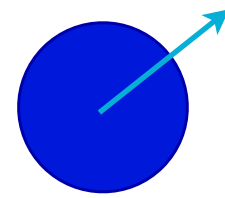


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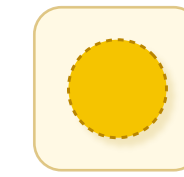
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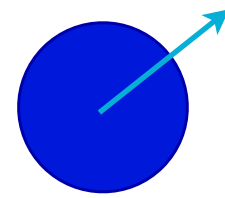
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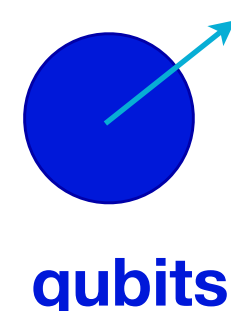
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qubits



febits

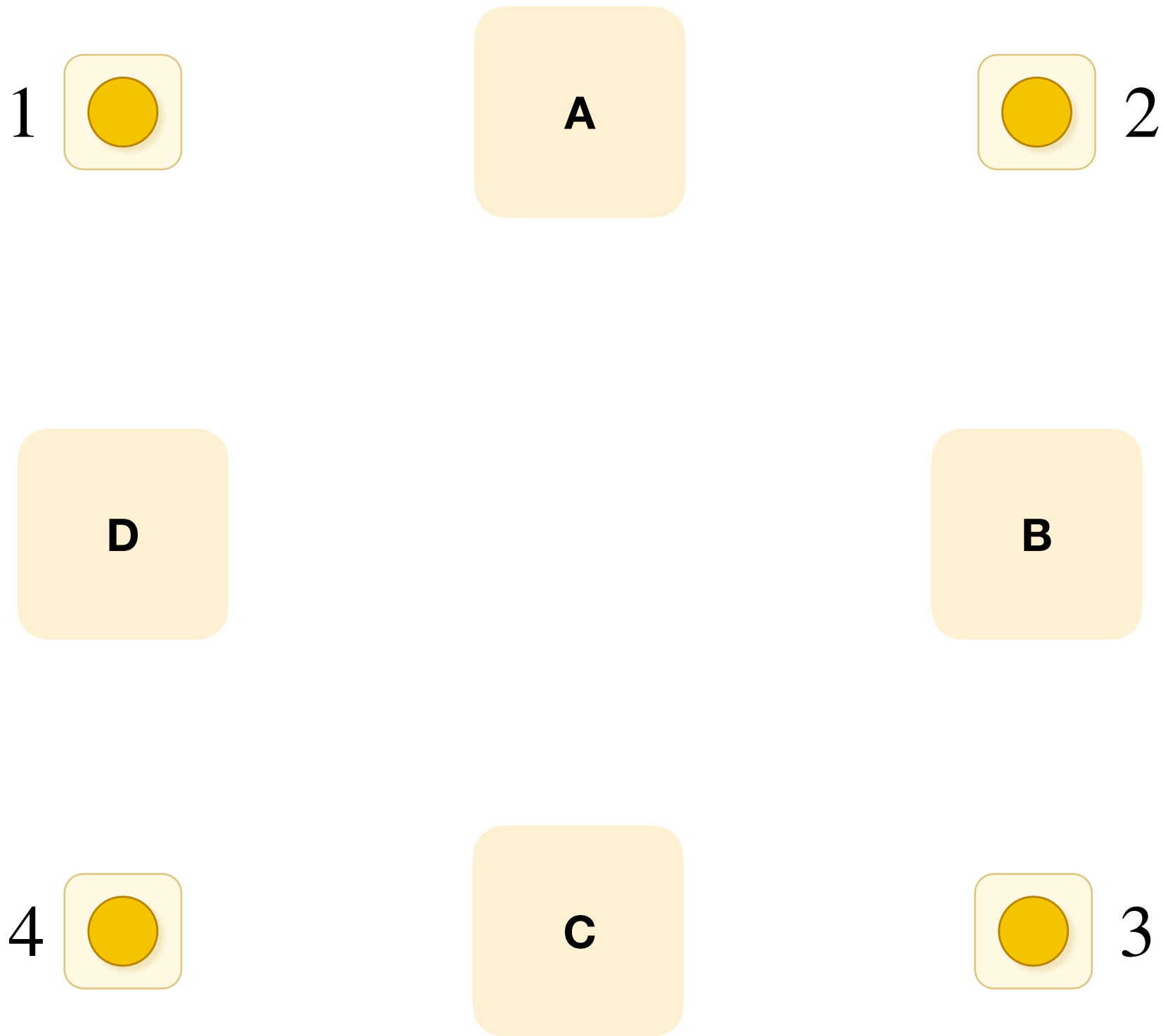
	Distinguishable particles	Bosons	Fermions
Via internal degree of freedom	Qubits	Qubits	Qubits
Via presence/absence	Qubits	Qubits	Febits

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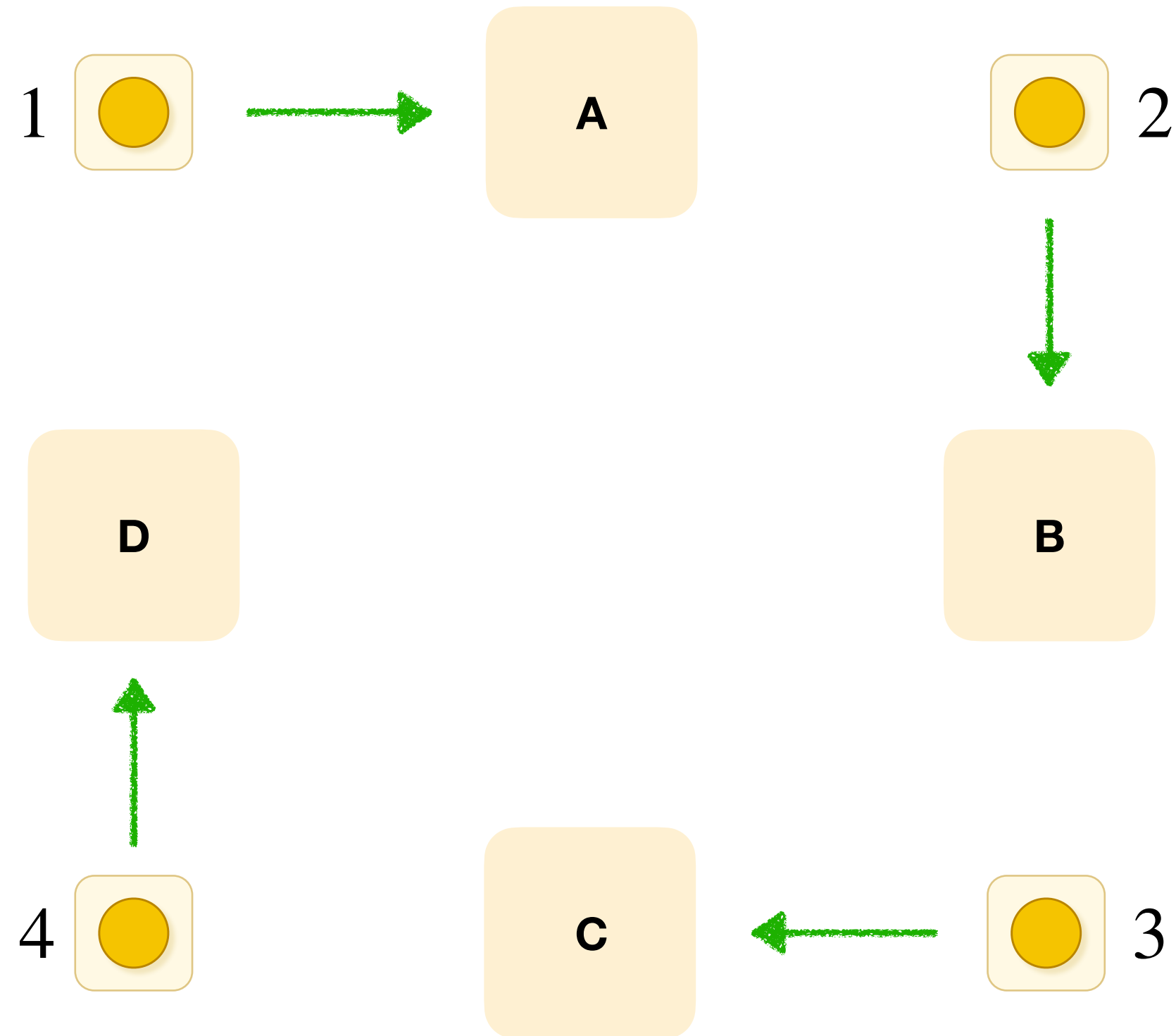
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Intuition for febits advantages



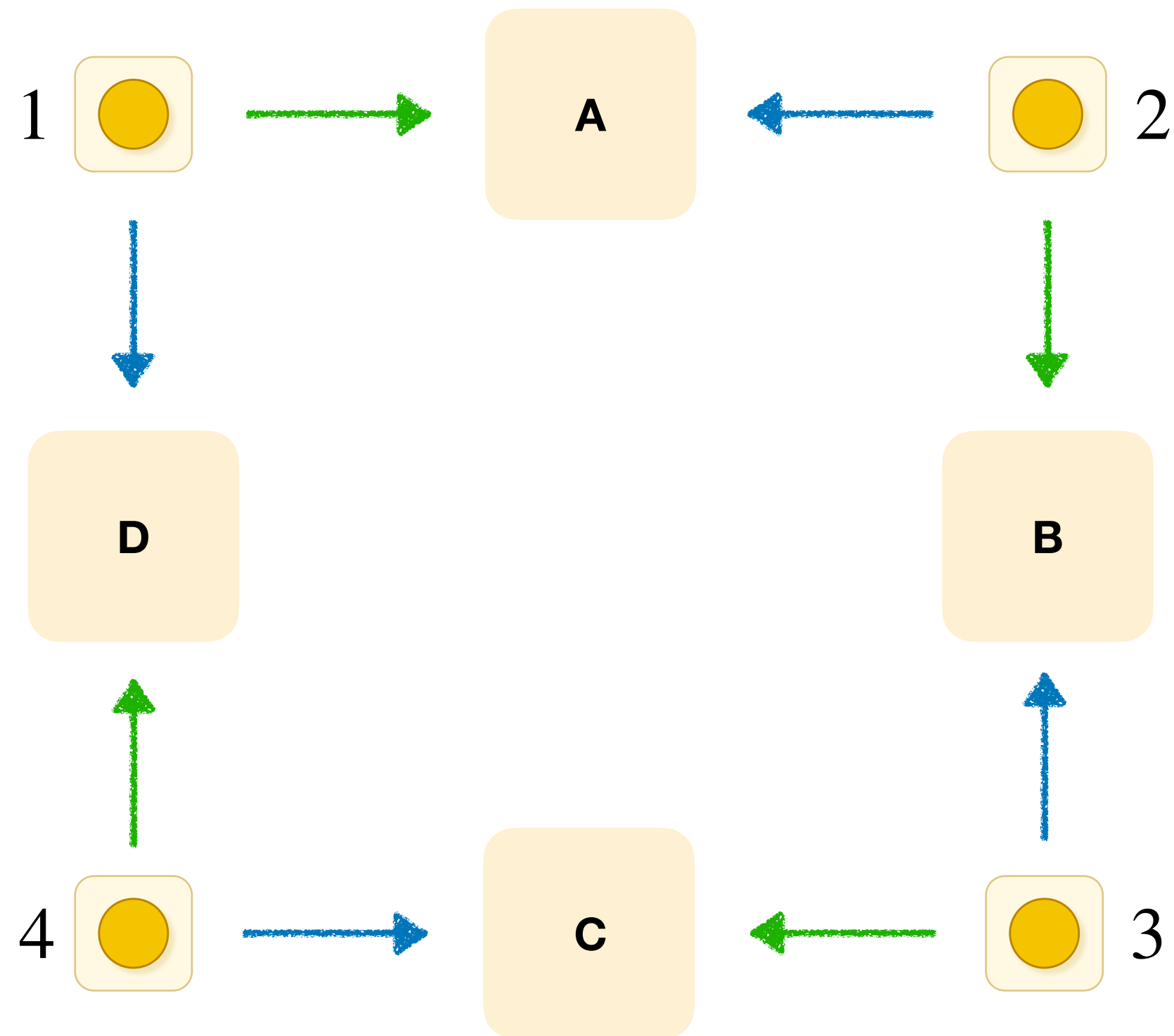
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- N febits all sent clockwise: $|\psi_c\rangle = |1\cdots 1\rangle_{1,2,\dots,N}$

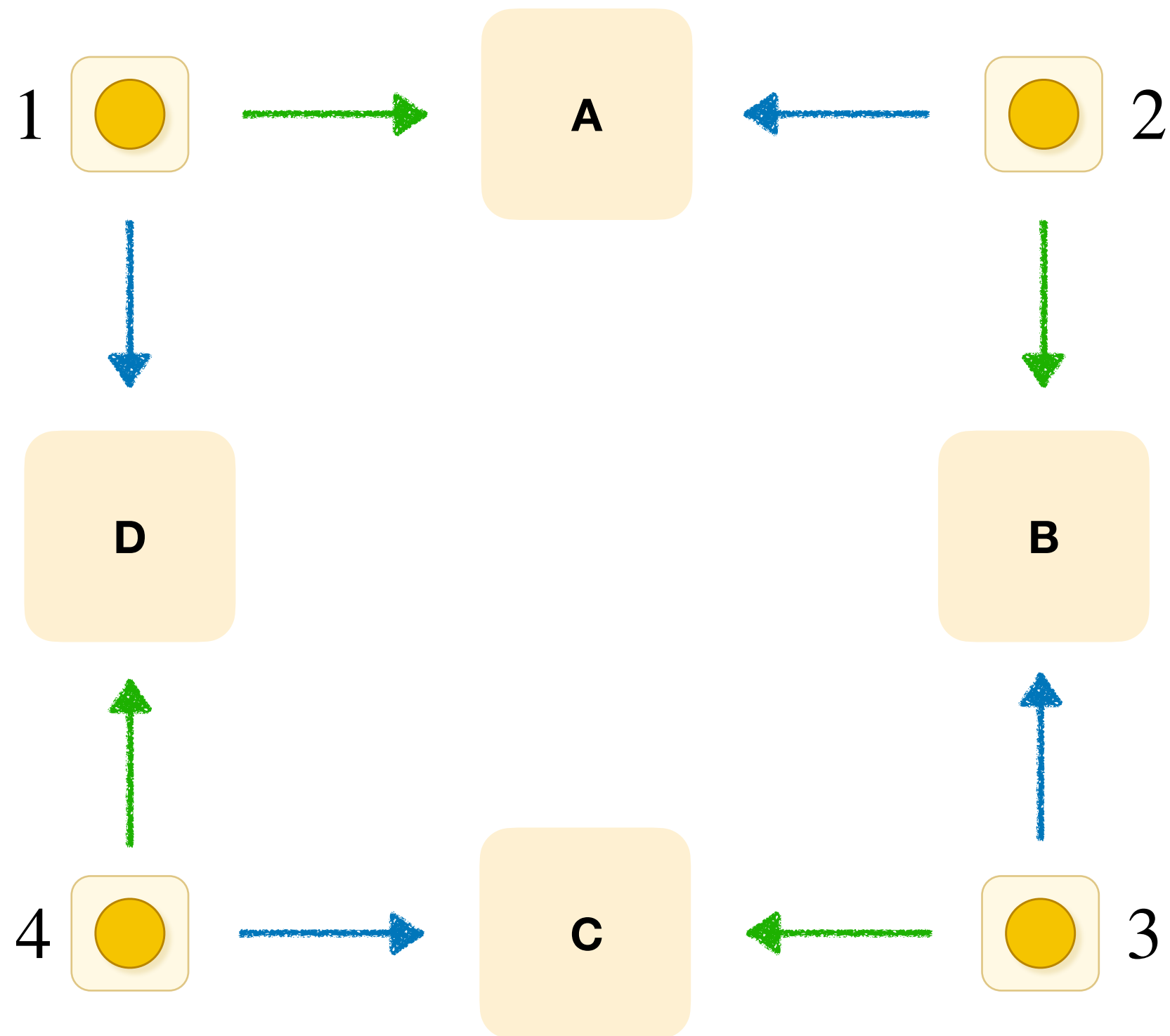


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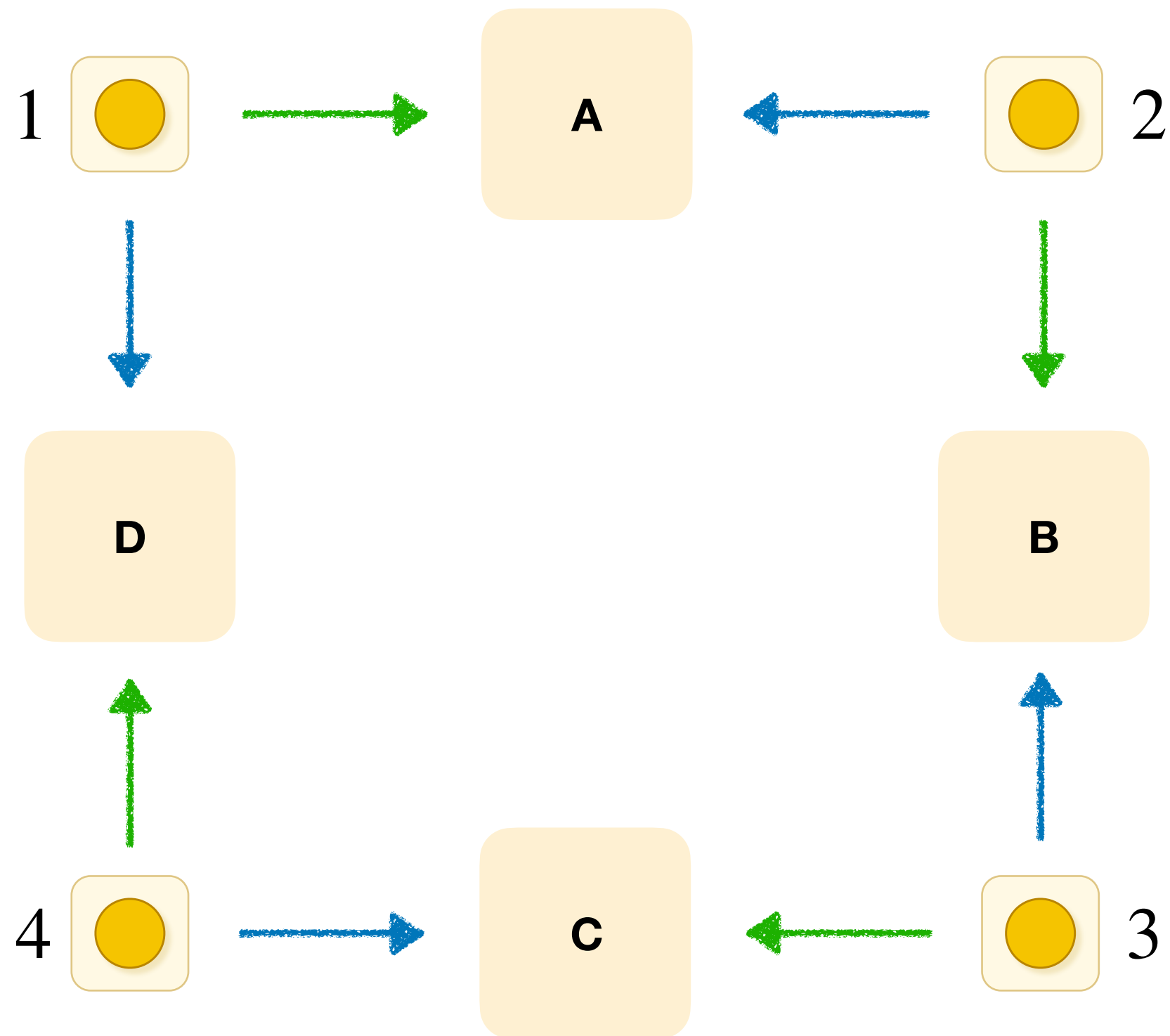


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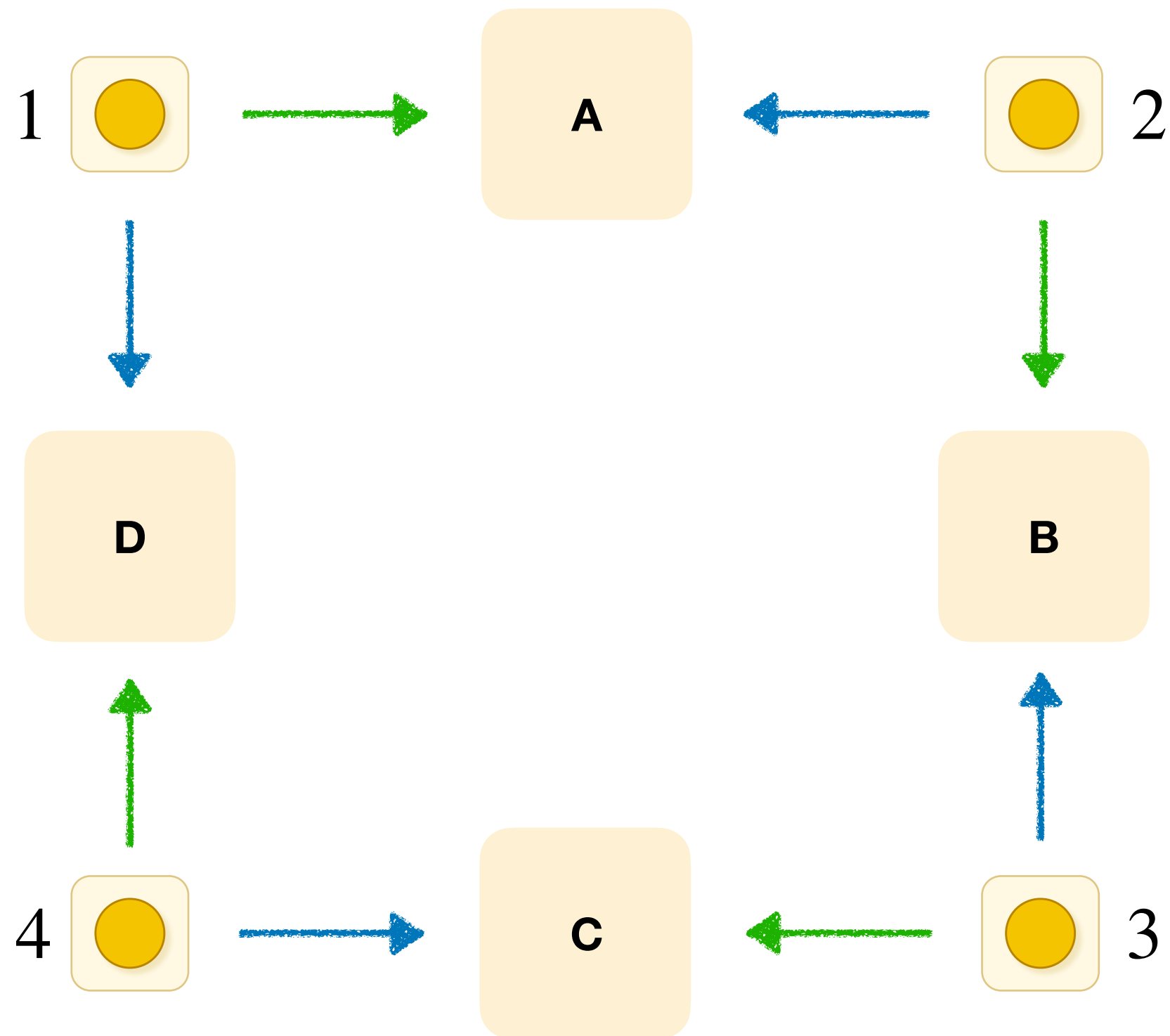
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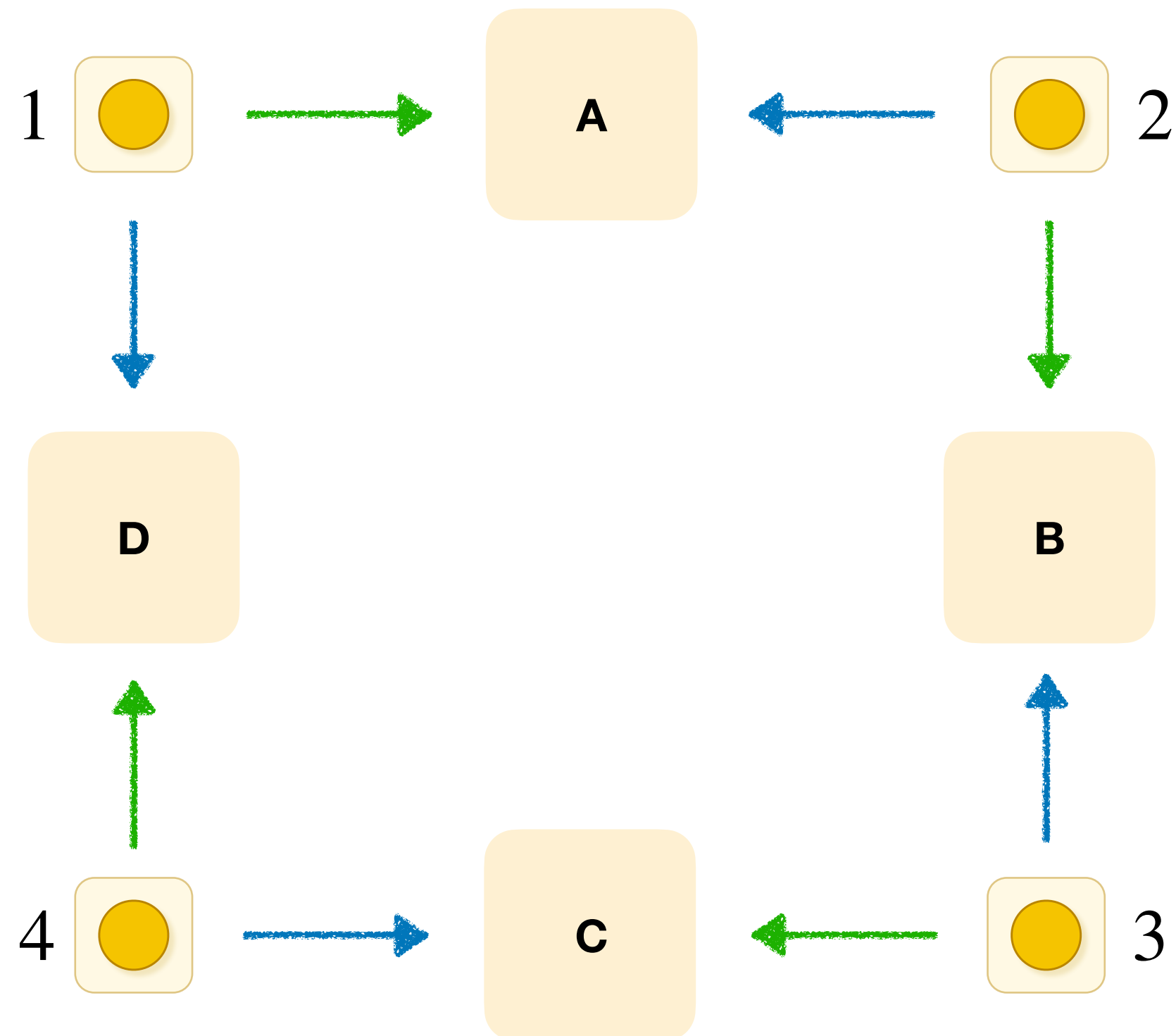
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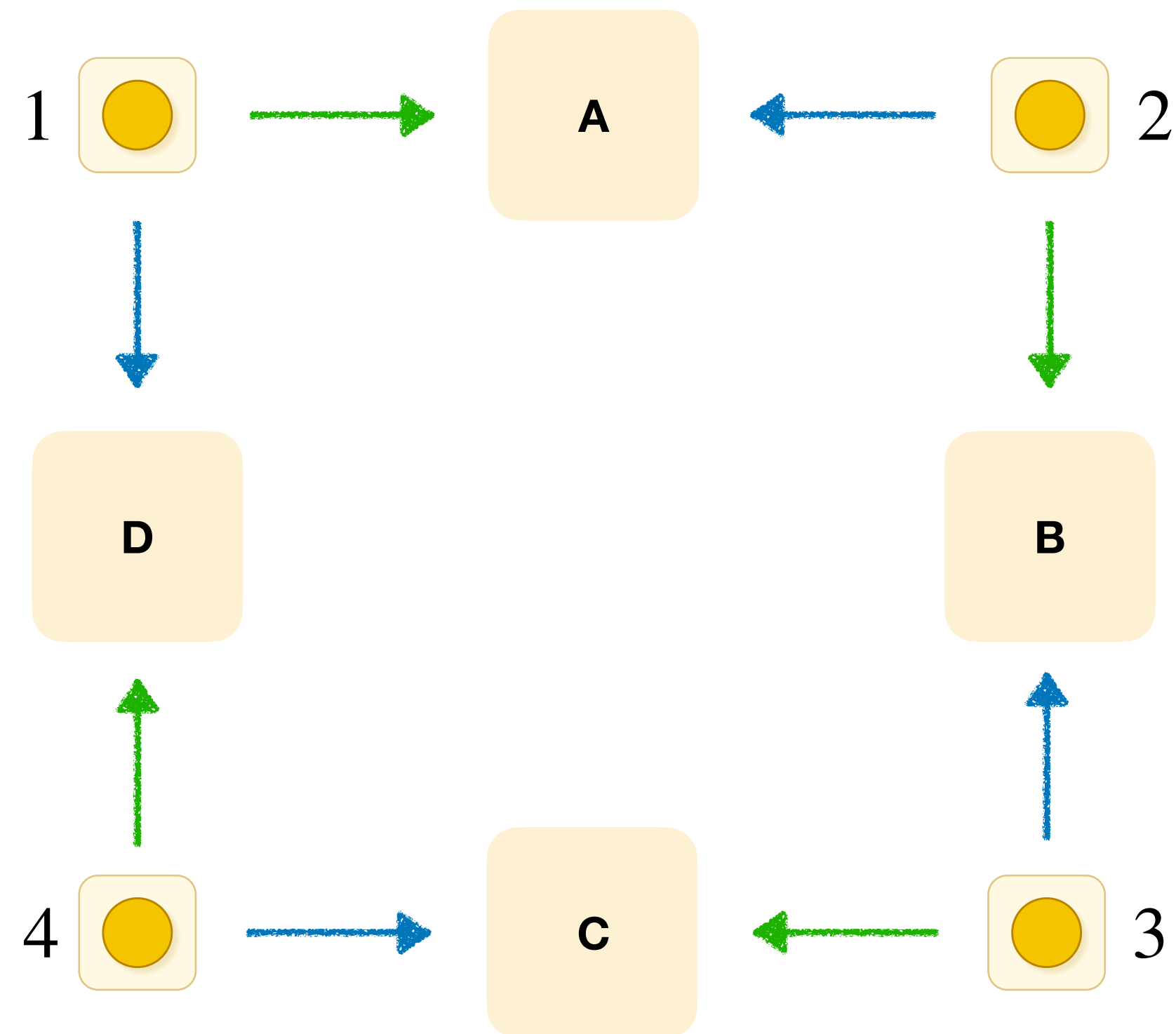
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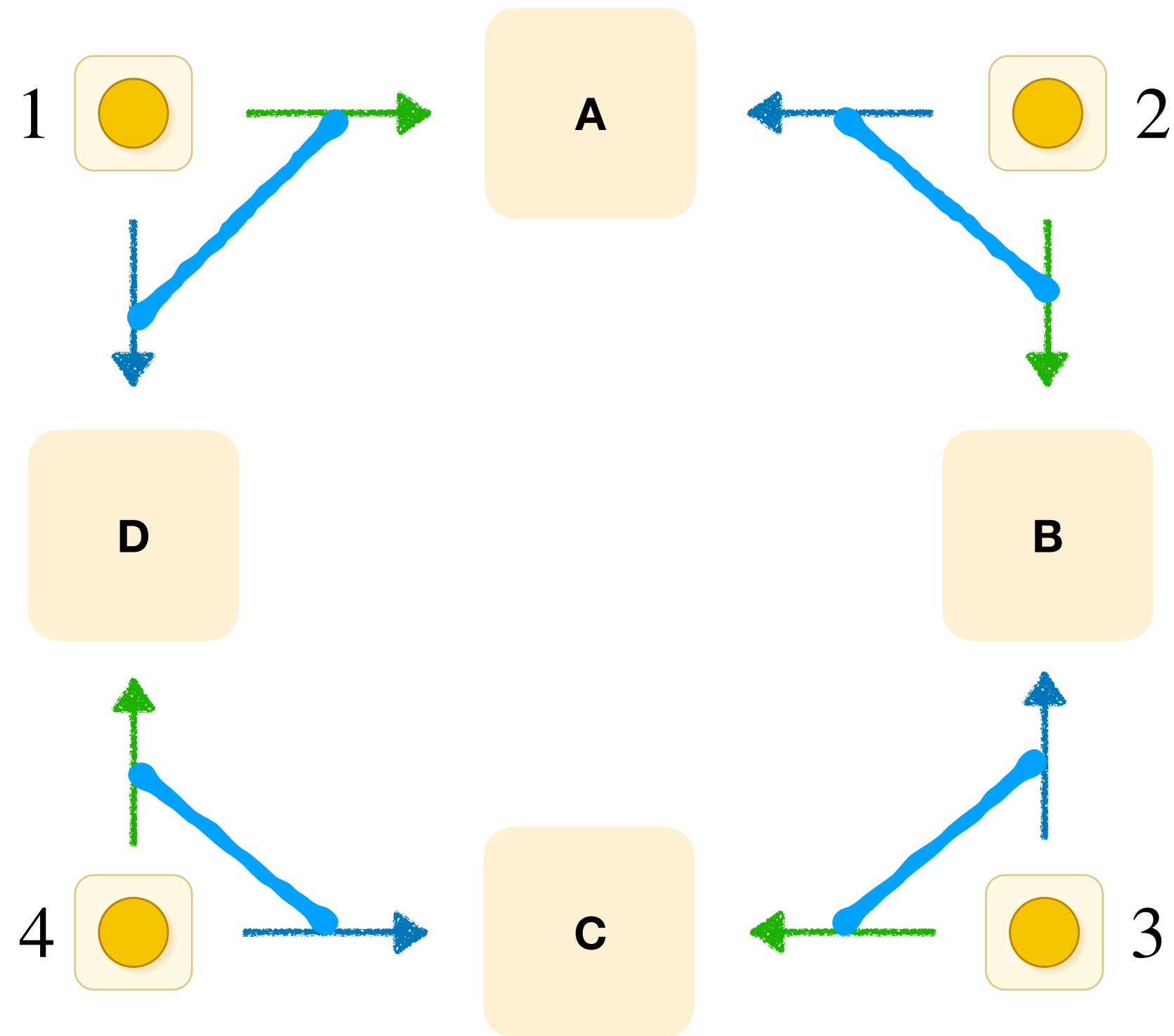
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Detecting $(-1)^{N-1}$



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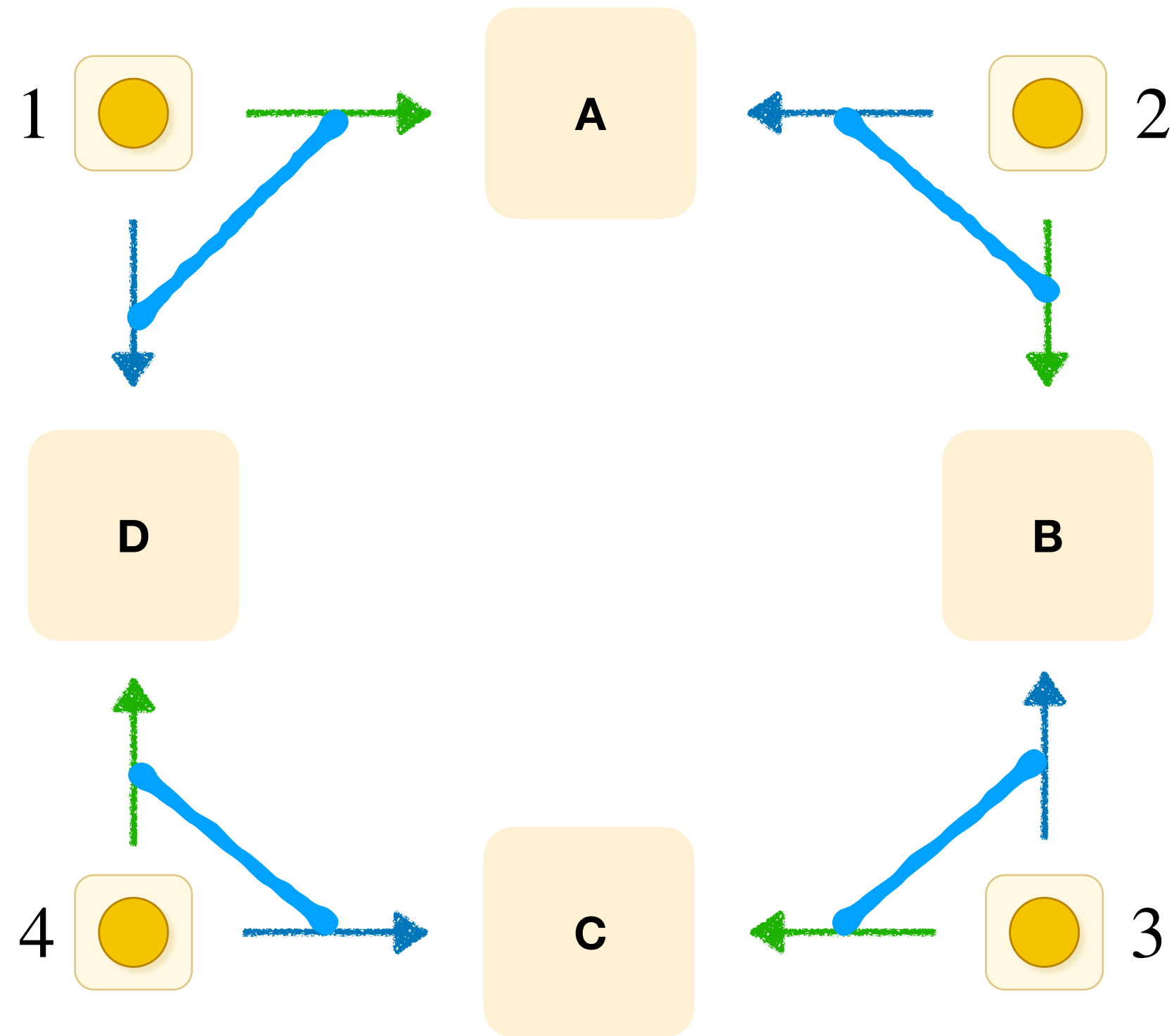
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- Superposition $(|\psi_a\rangle + |\psi_c\rangle)/\sqrt{2}$ for a detectable relative phase!

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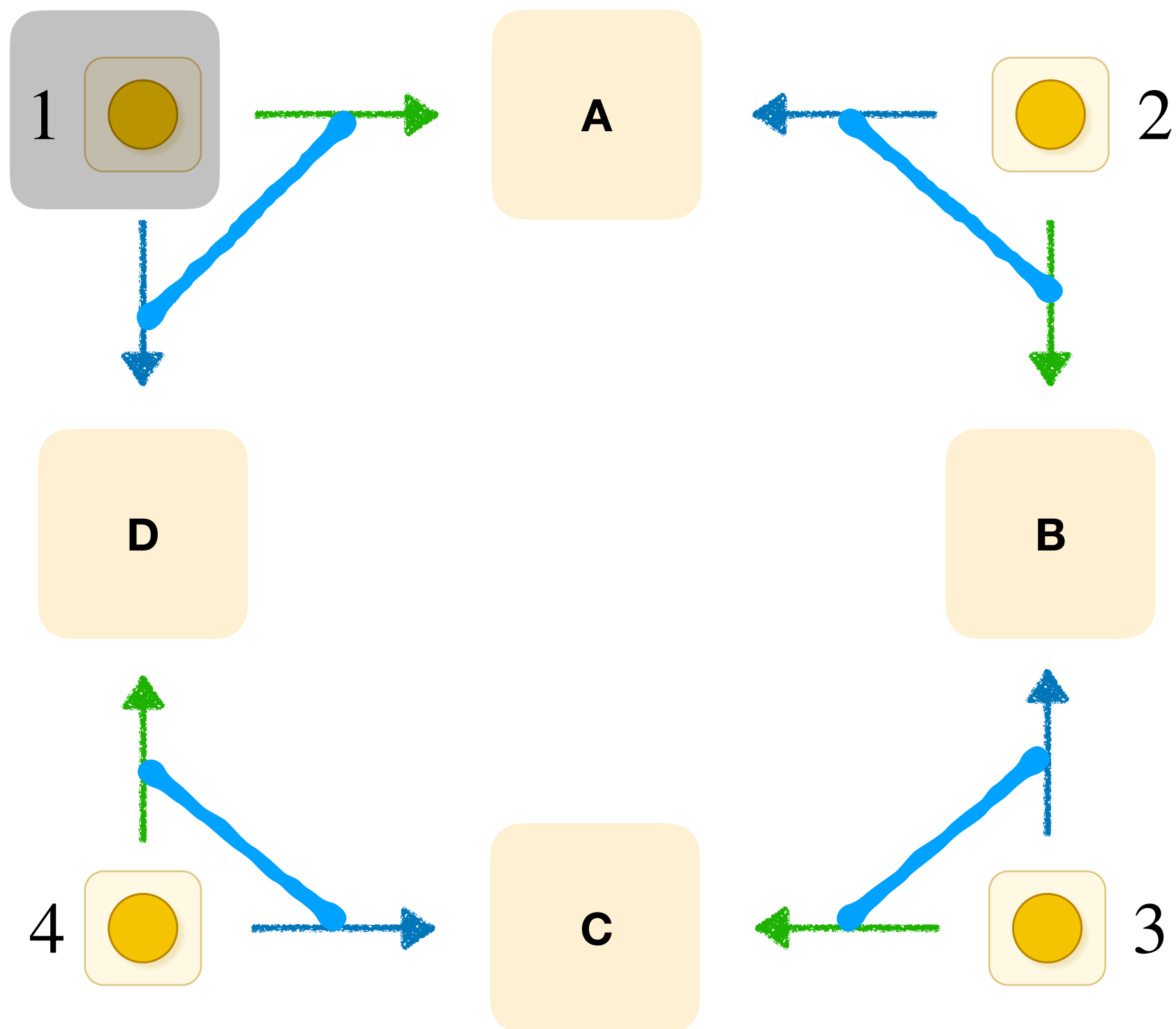
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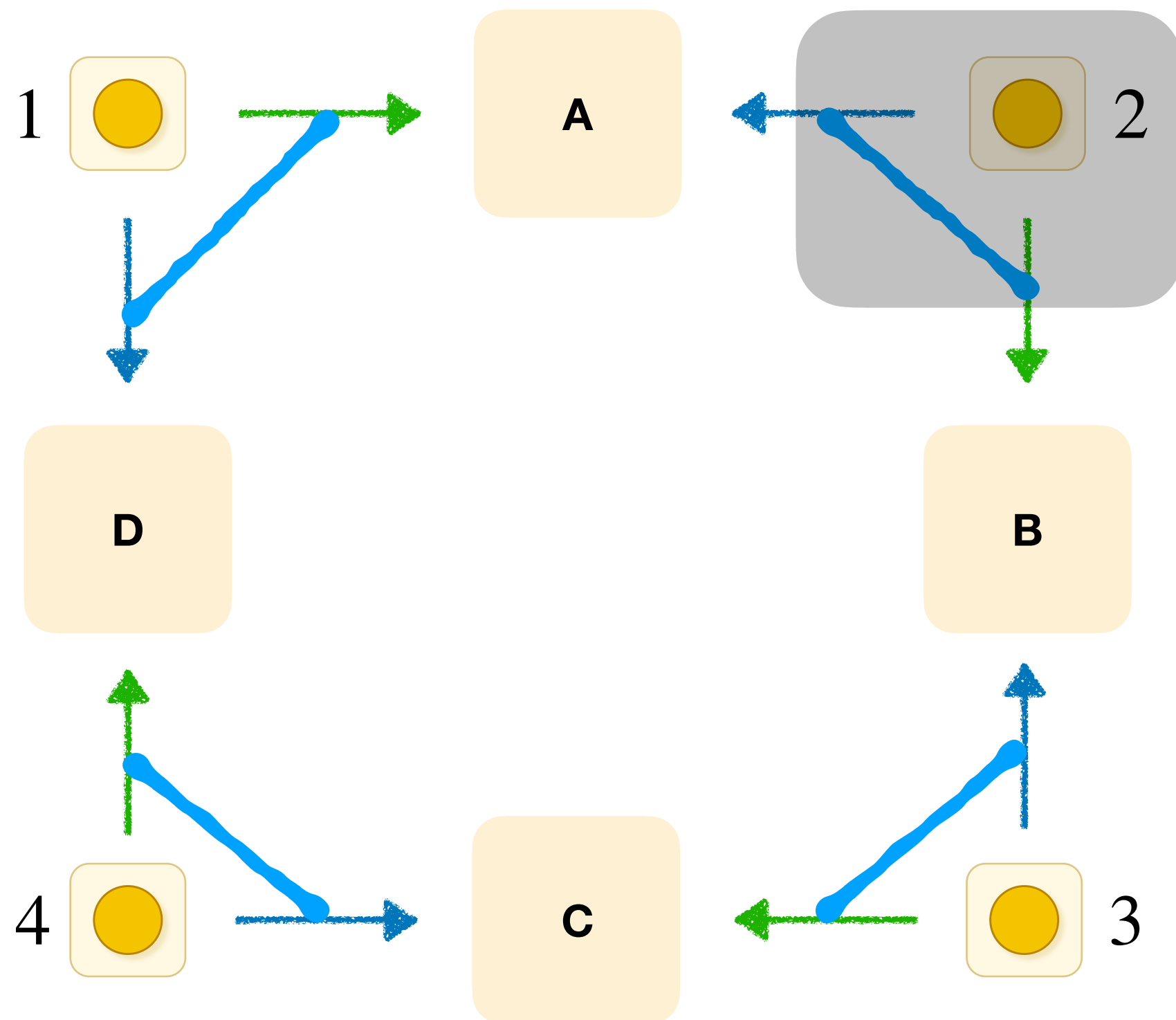
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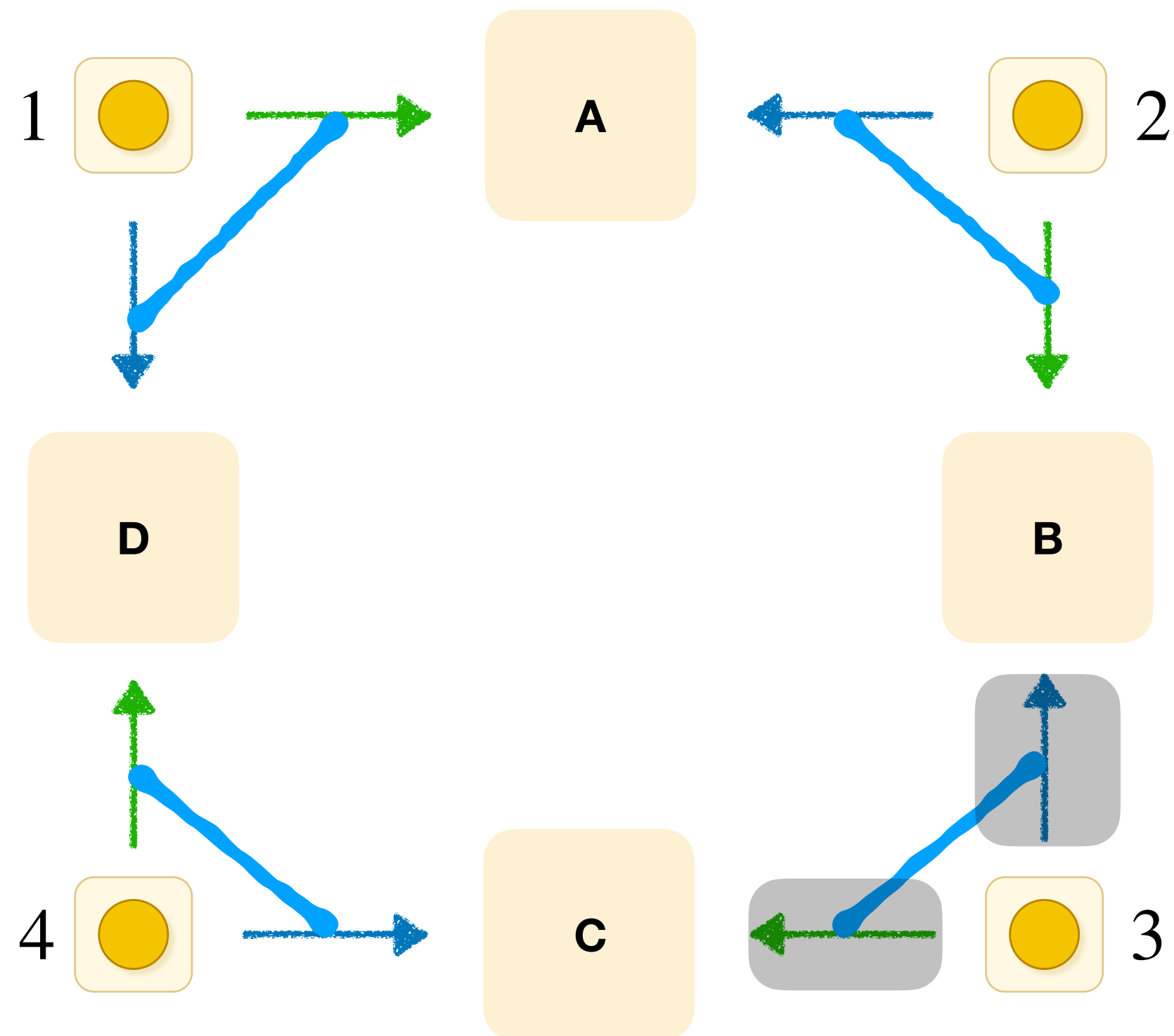


- Superposition $(|\psi_a\rangle + |\psi_c\rangle)/\sqrt{2}$ for a detectable relative phase!
- For experiment-minded physicists:
 1. Initialize with cold atoms with frozen degrees of freedom
 2. Fermionic beam splitter by tunneling between optical traps

[González-Cuadra et al., *Fermionic quantum processing with programmable neutral atom arrays*, 2023]

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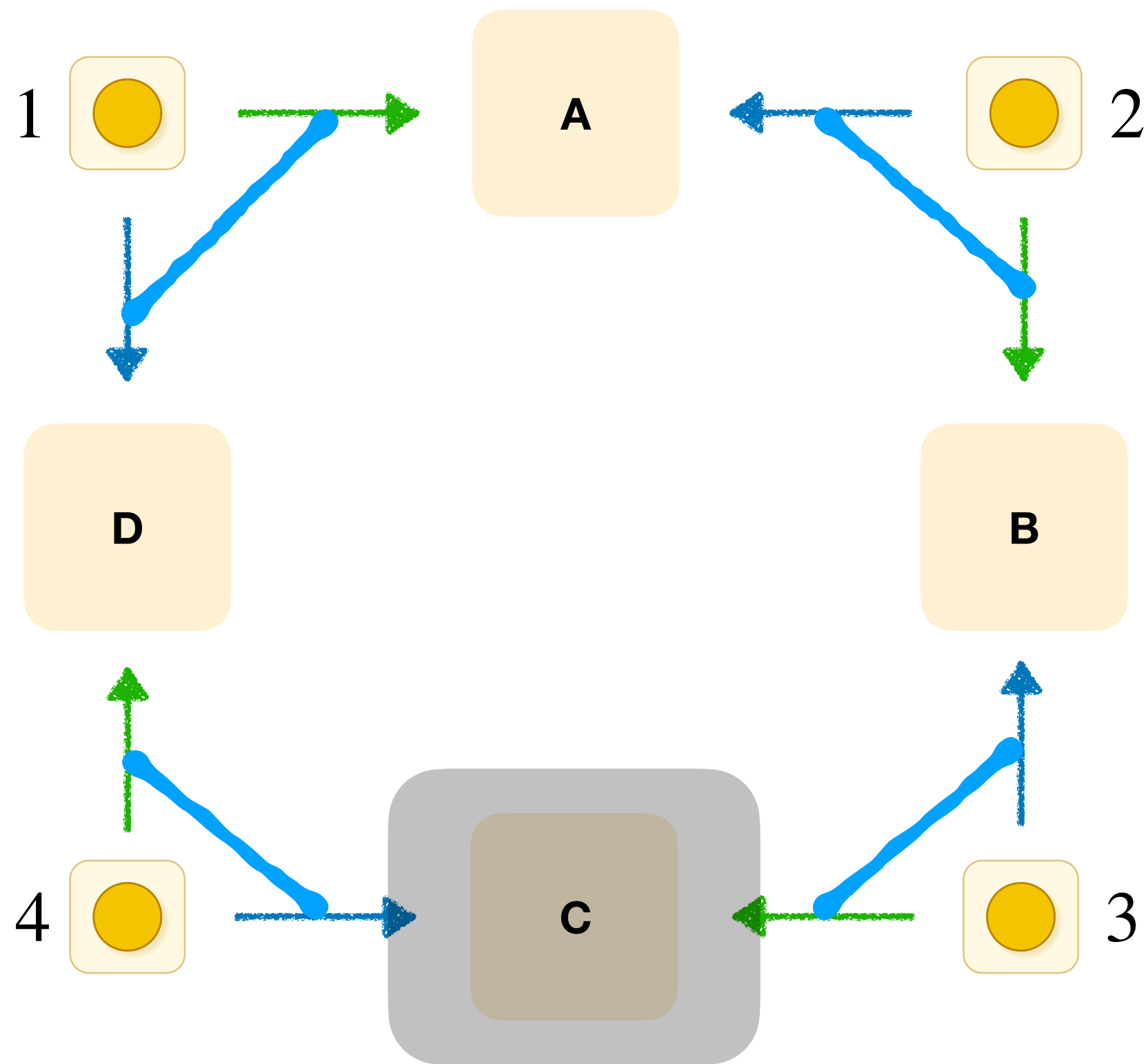
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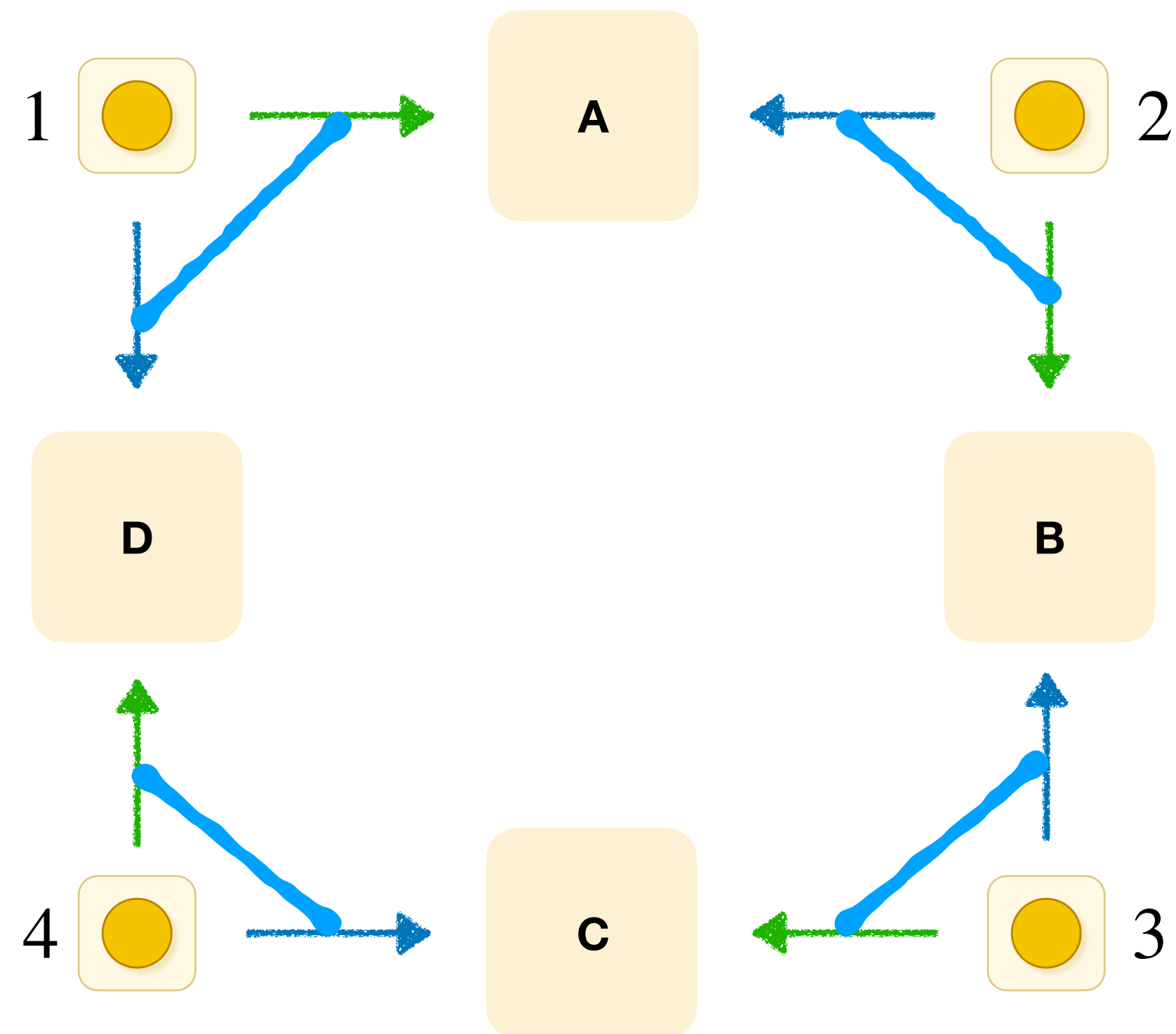


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 4. Measurements with fermionic linear optics

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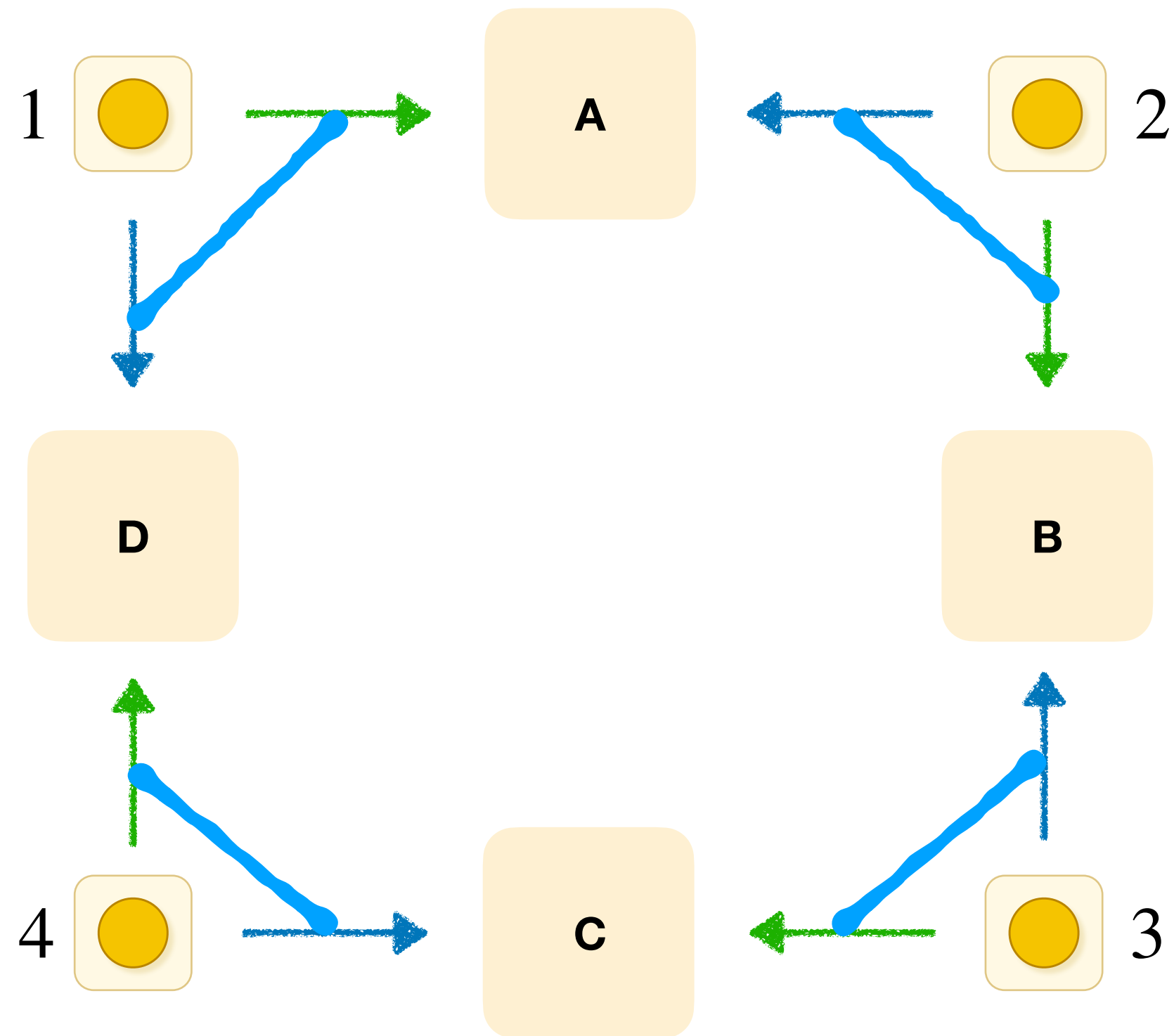
Distributed quantum network to hide $(-1)^{N-1}$



Intuition for febits advantages

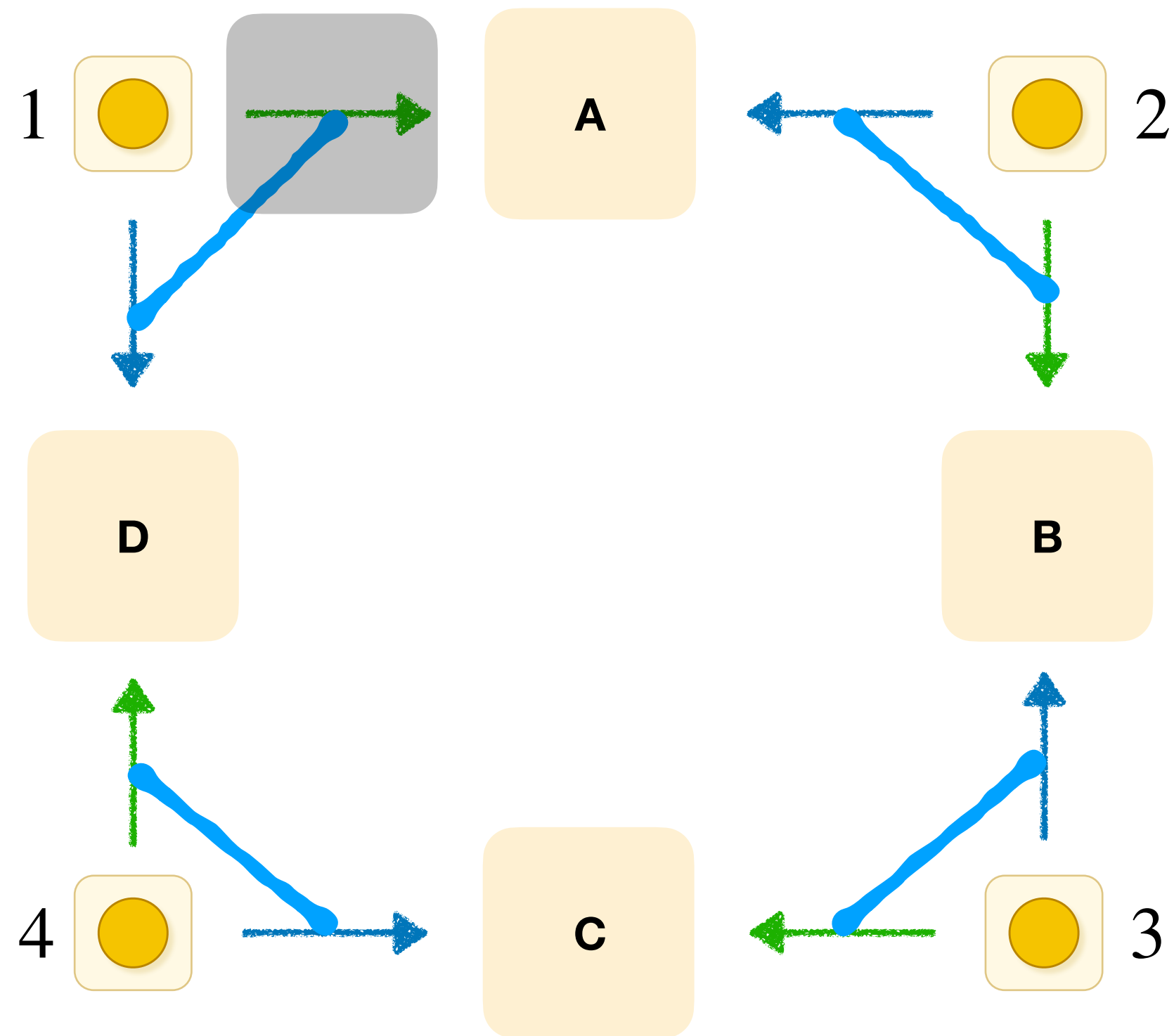
Distributed quantum network to hide $(-1)^{N-1}$

- $(|\psi_a\rangle + |\psi_c\rangle)/\sqrt{2}$ with detectable relative phase



Intuition for febits advantages

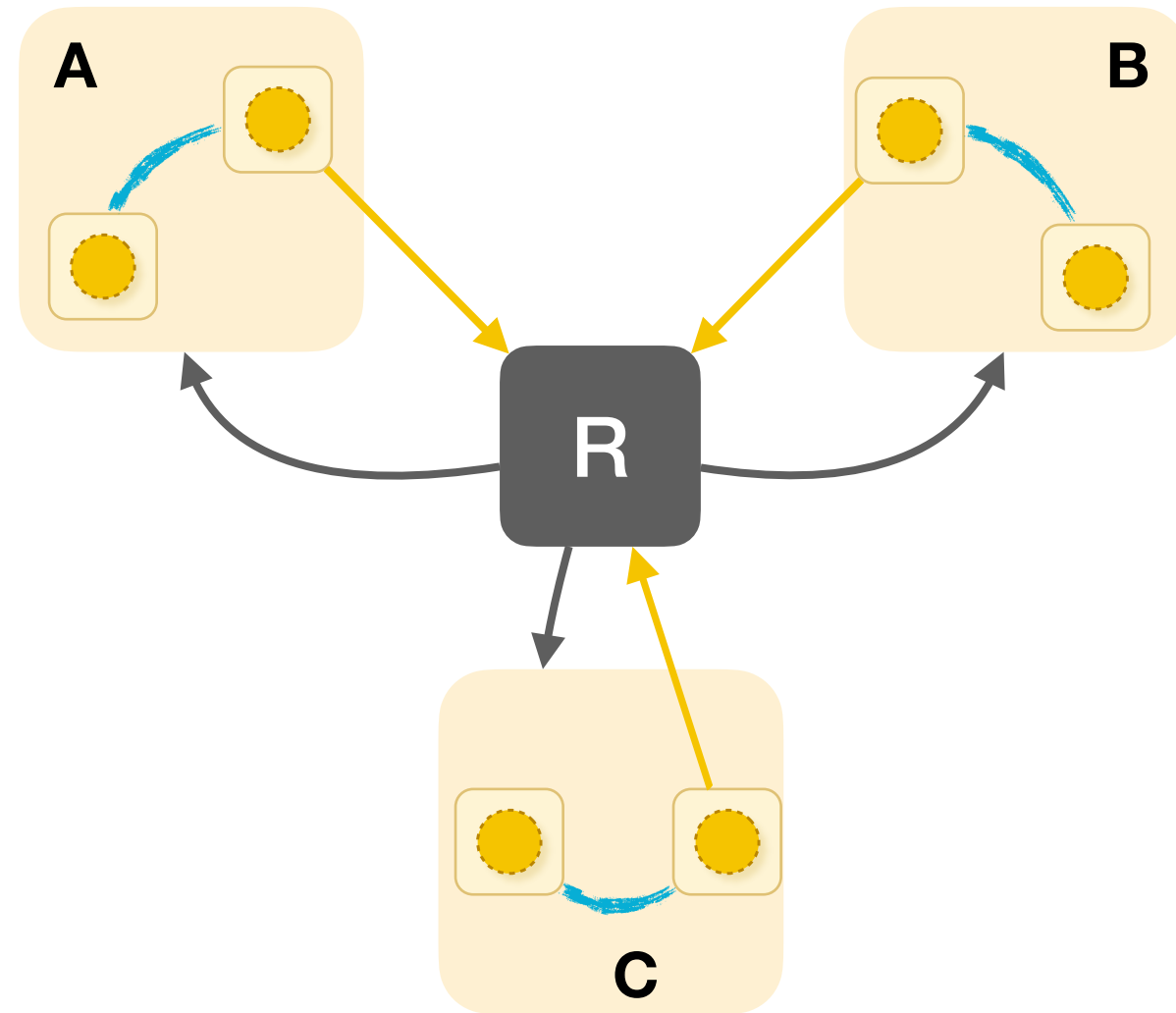
Distributed quantum network to hide $(-1)^{N-1}$



- $(|\psi_a\rangle + |\psi_c\rangle)/\sqrt{2}$ with detectable relative phase
- But qubits can simulate $(|\psi_a\rangle + |\psi_c\rangle)/\sqrt{2}$ in one step by explicitly introducing $(-1)^{N-1}$?

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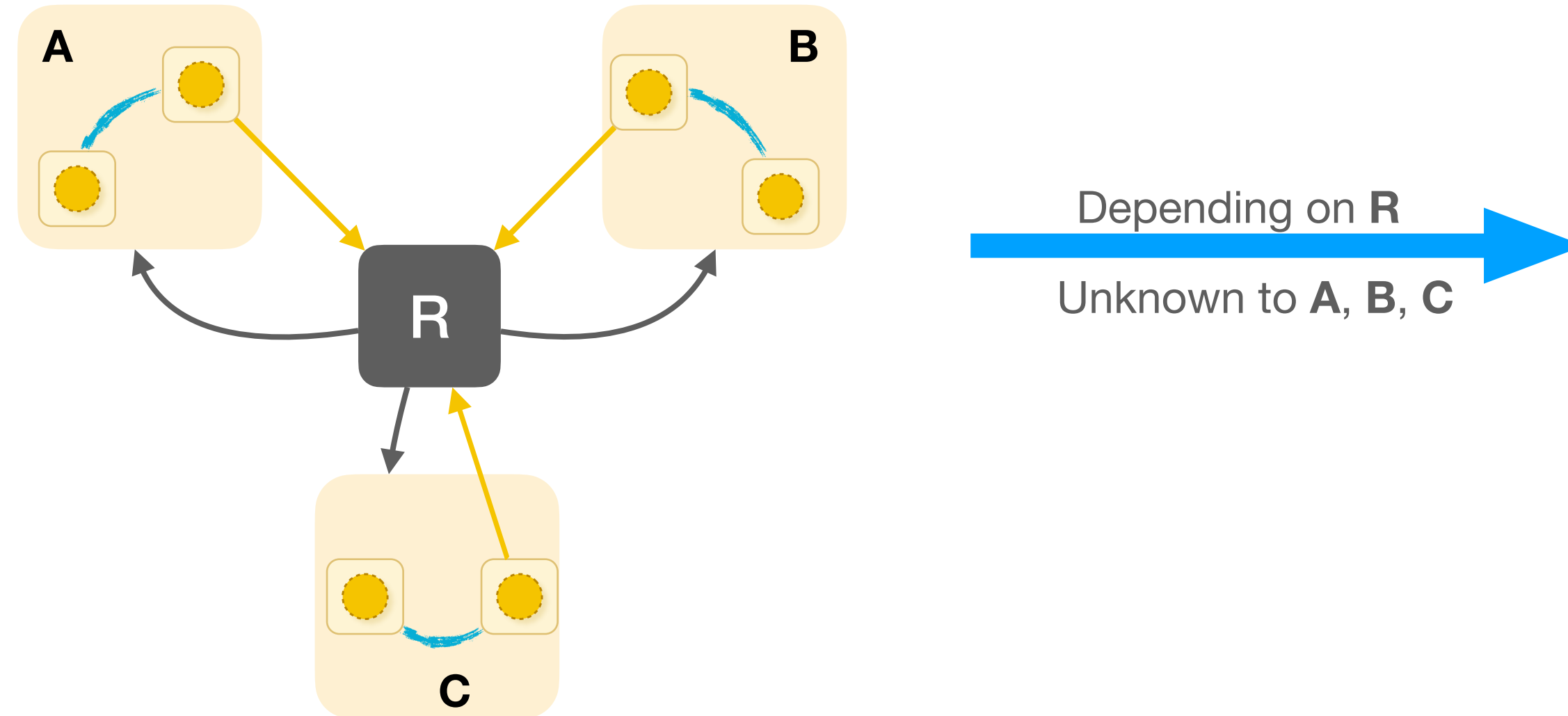
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- $(|\psi_a\rangle + |\psi_c\rangle)/\sqrt{2}$ with detectable relative phase
- But qubits can simulate $(|\psi_a\rangle + |\psi_c\rangle)/\sqrt{2}$ in one step by explicitly introducing $(-1)^{N-1}$?
- Varying and hiding N with a router **R**!

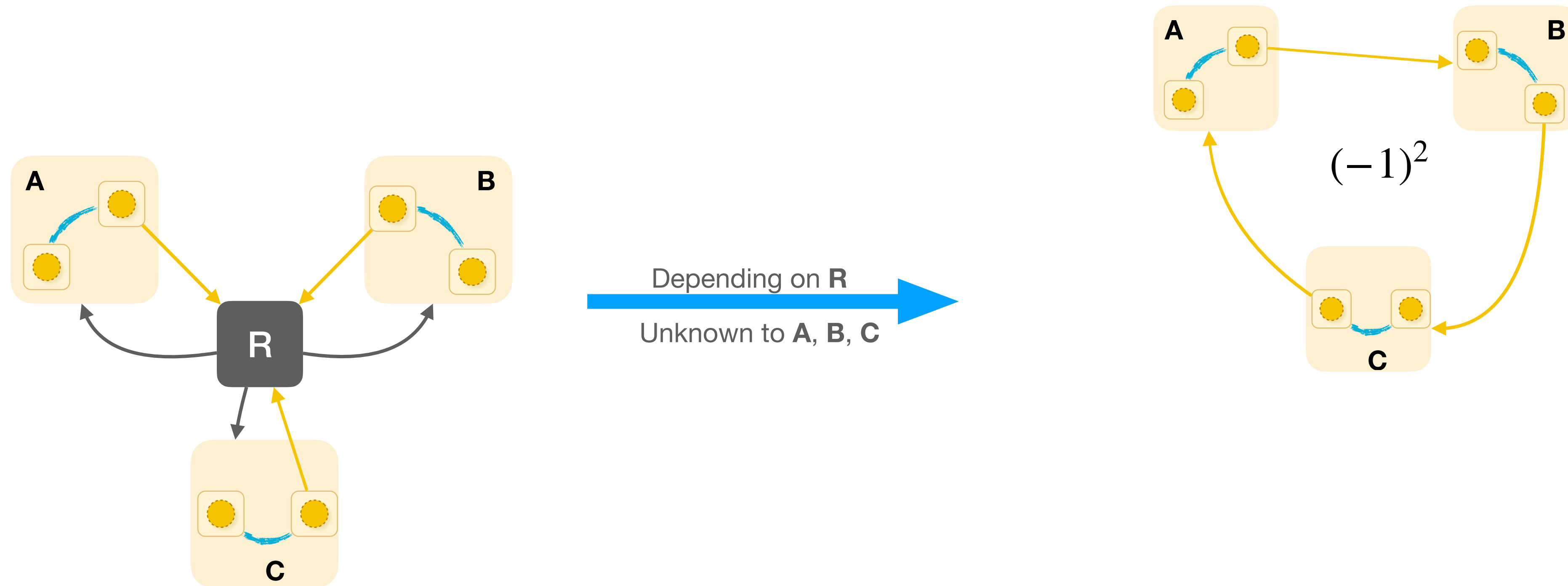
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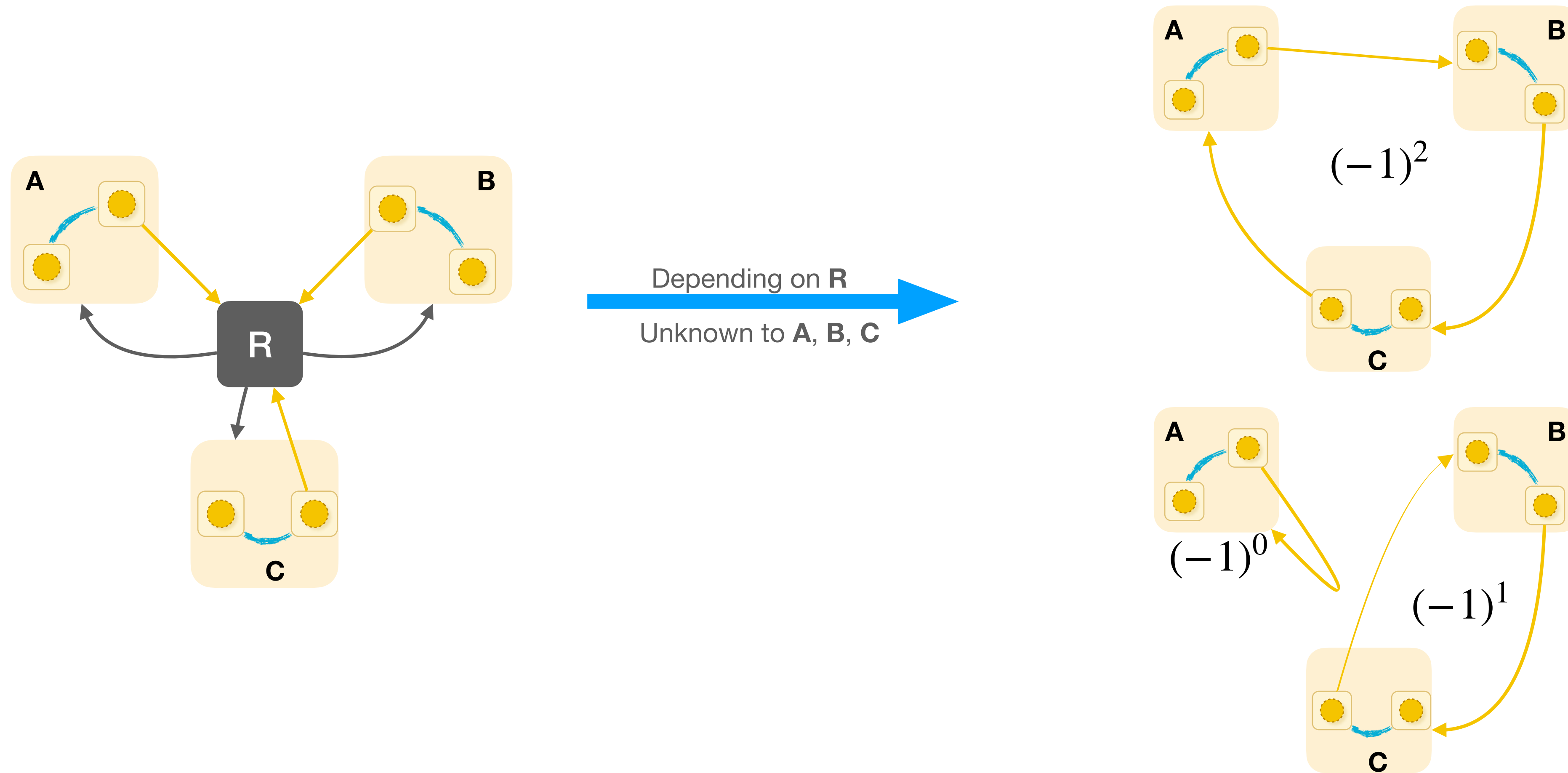
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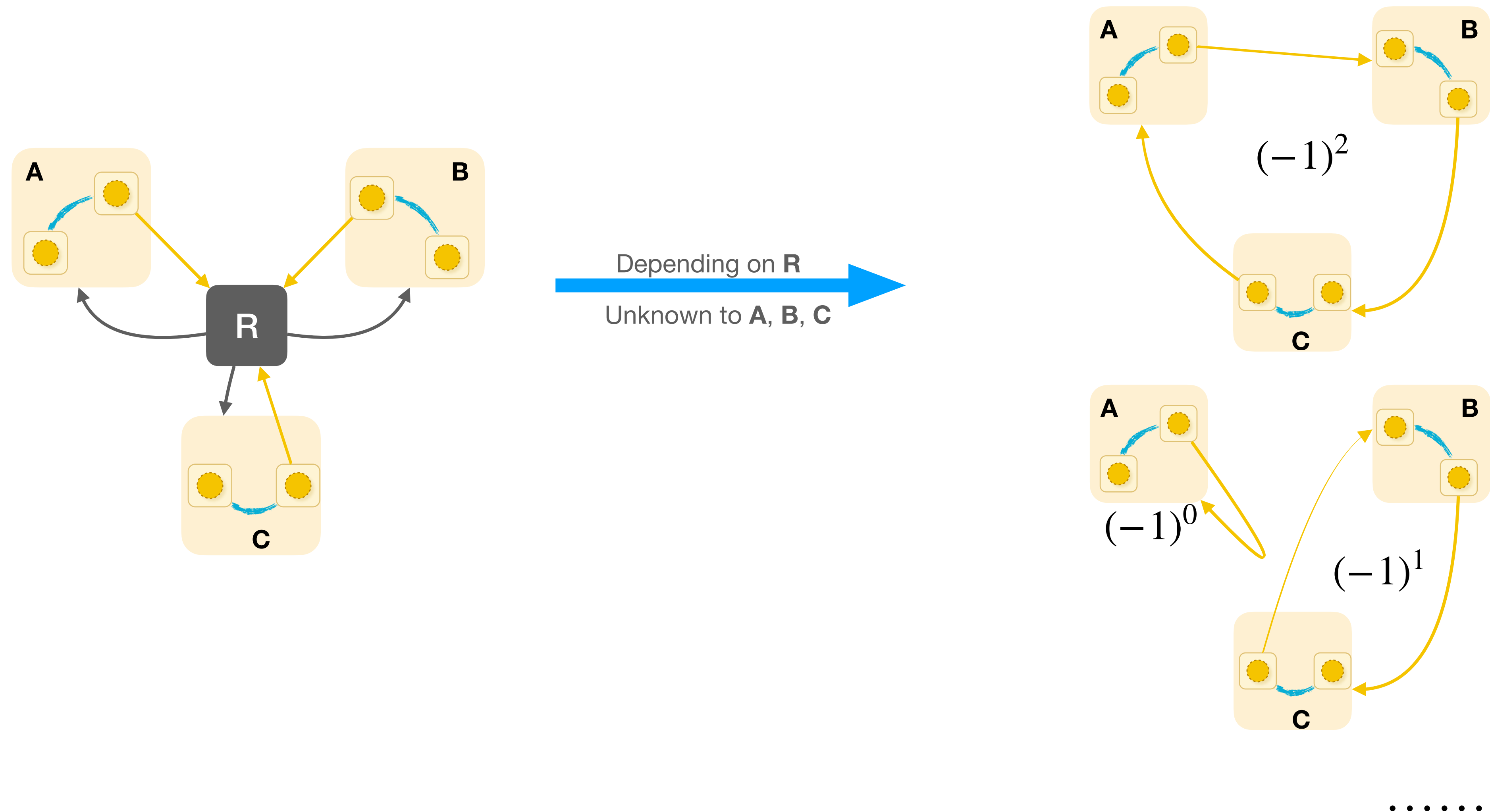
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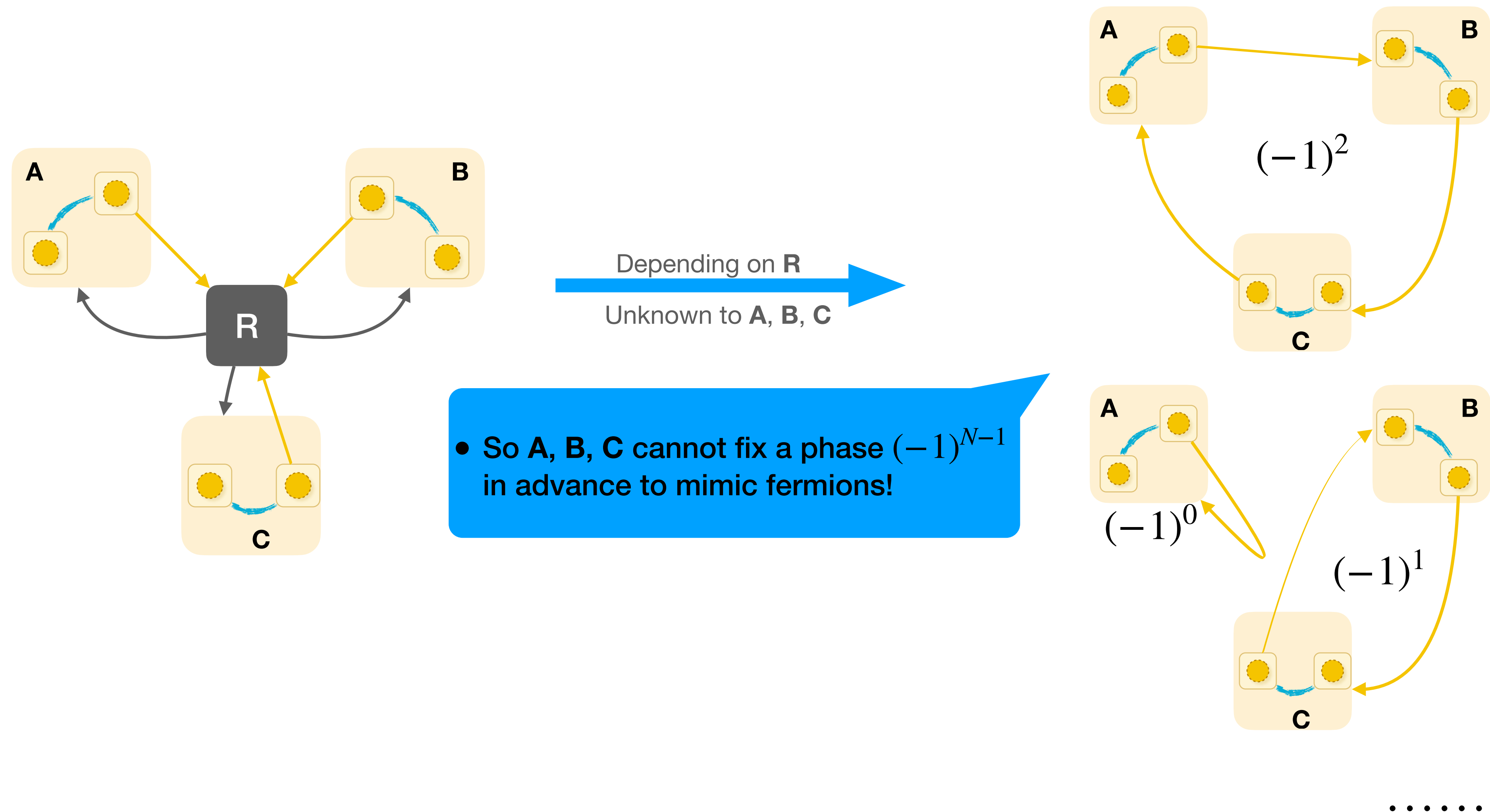
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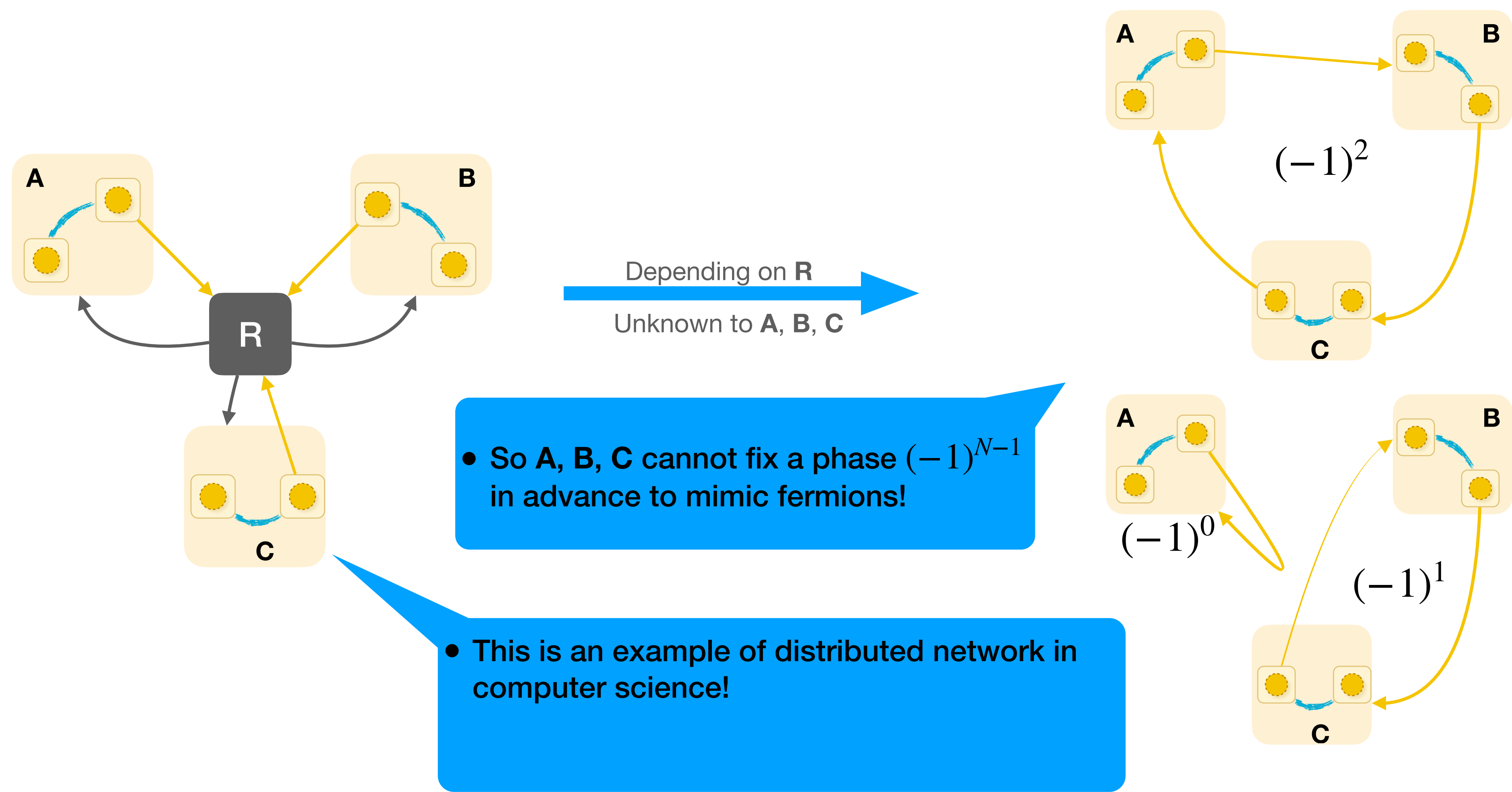
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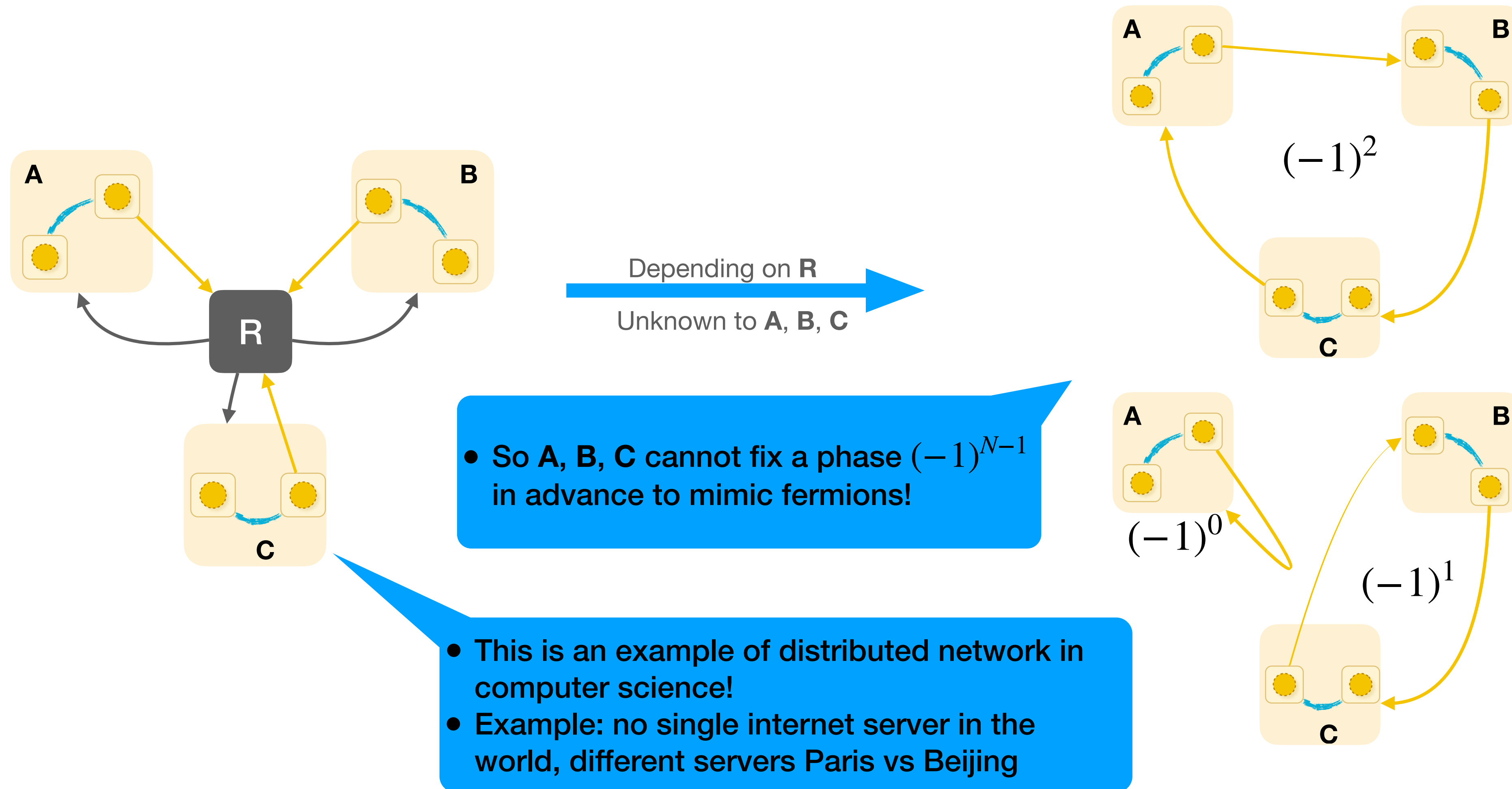
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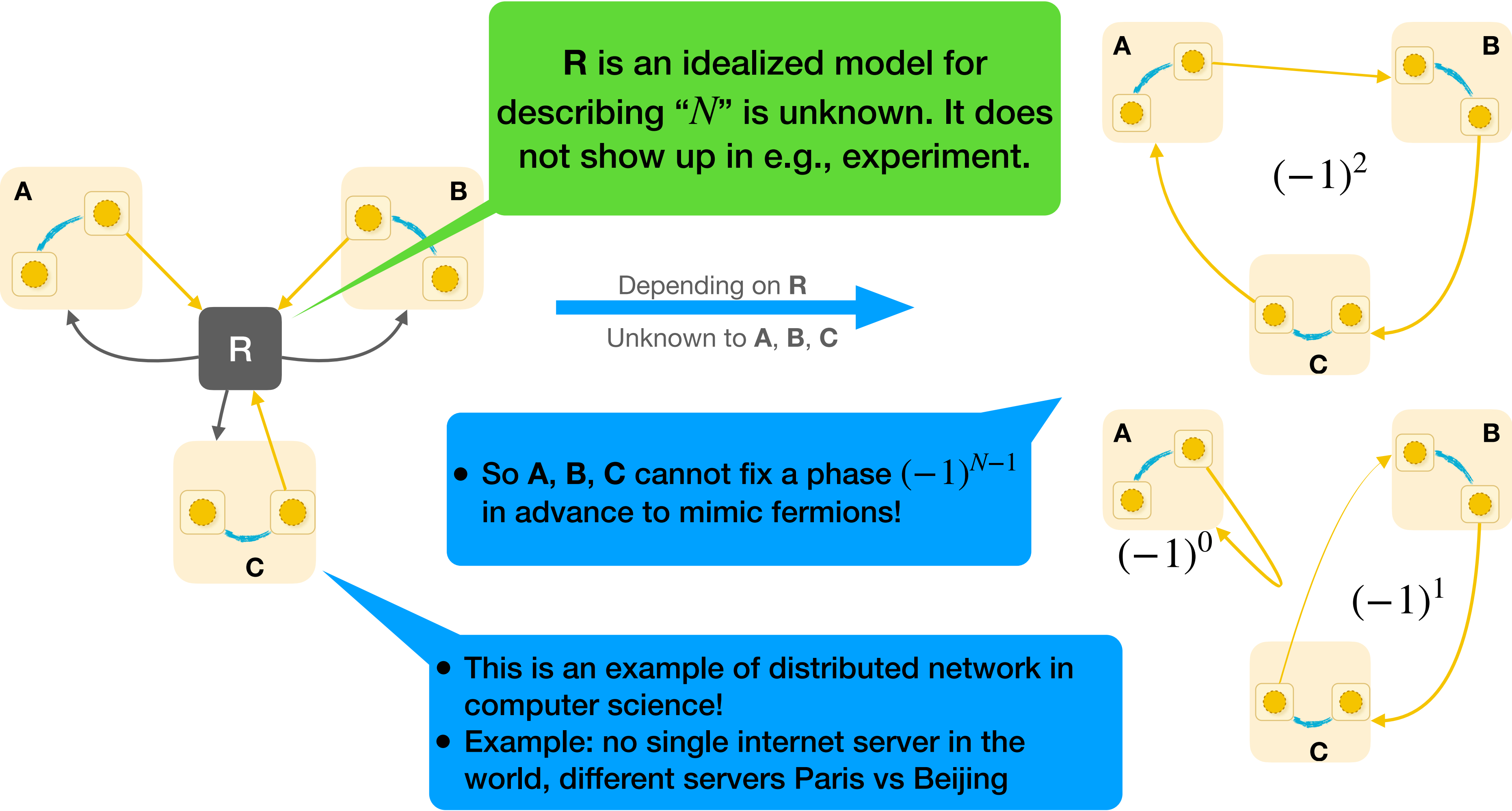
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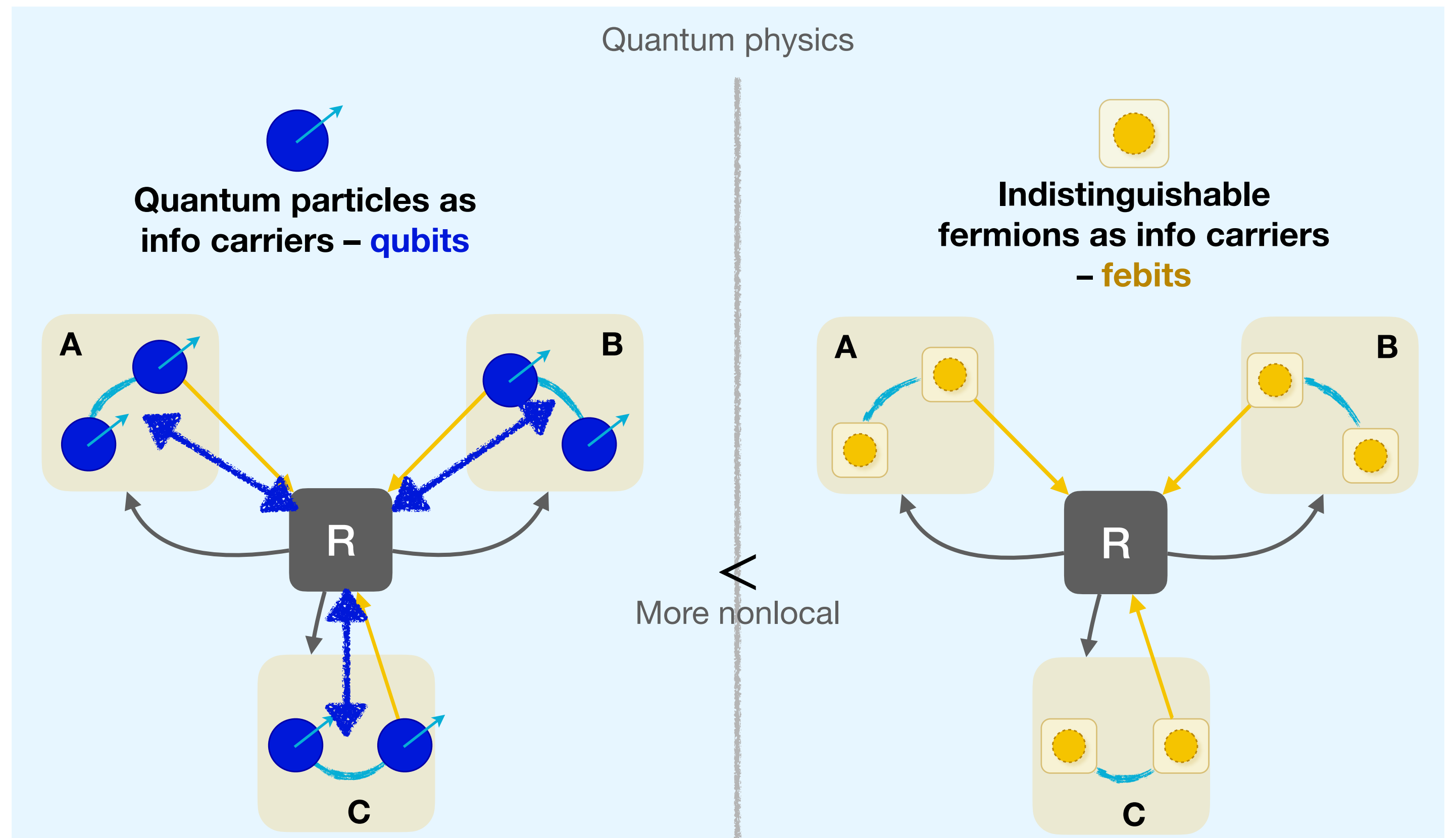
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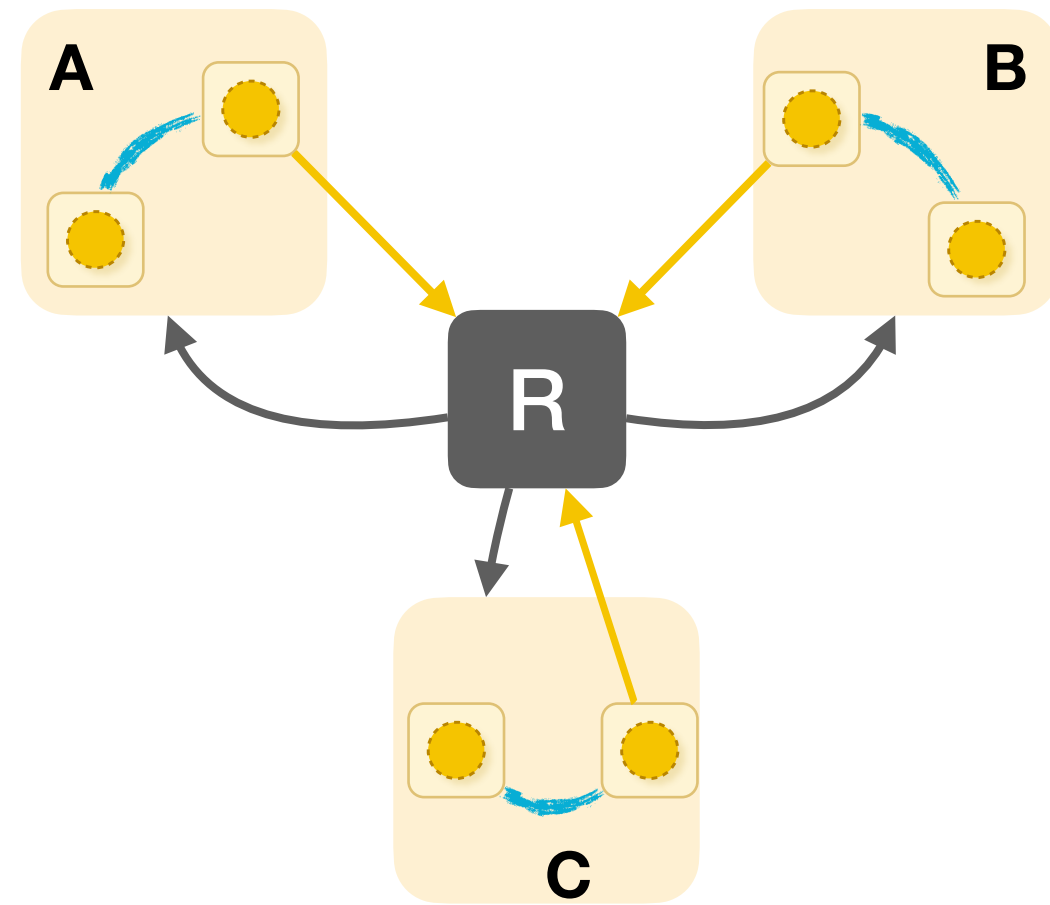
Outline

- What are febits
- Intuition for febit advantage
 1. Detecting exchange phase
 2. Distributed quantum network
- The Gedankenexperiment
- Proof: self-testing of qubits
- Conclusions



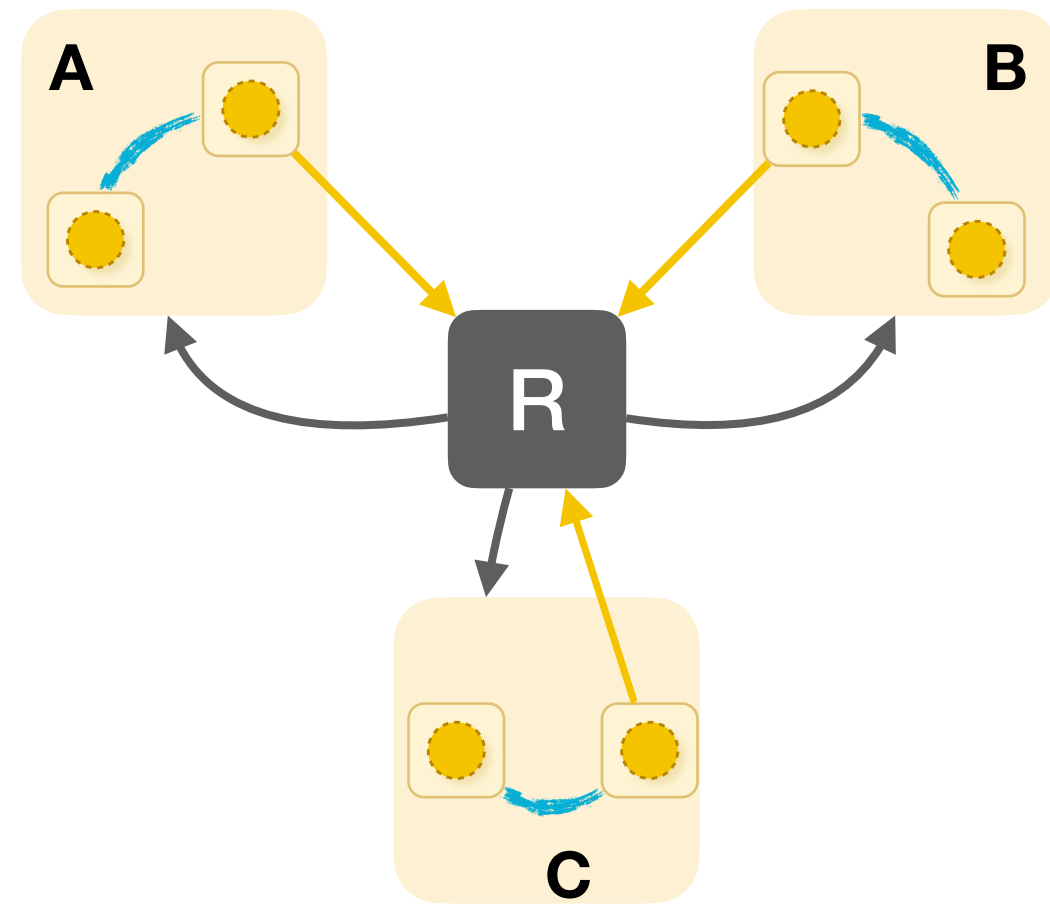
The Gedankenexperiment

Rule



The Gedankenexperiment

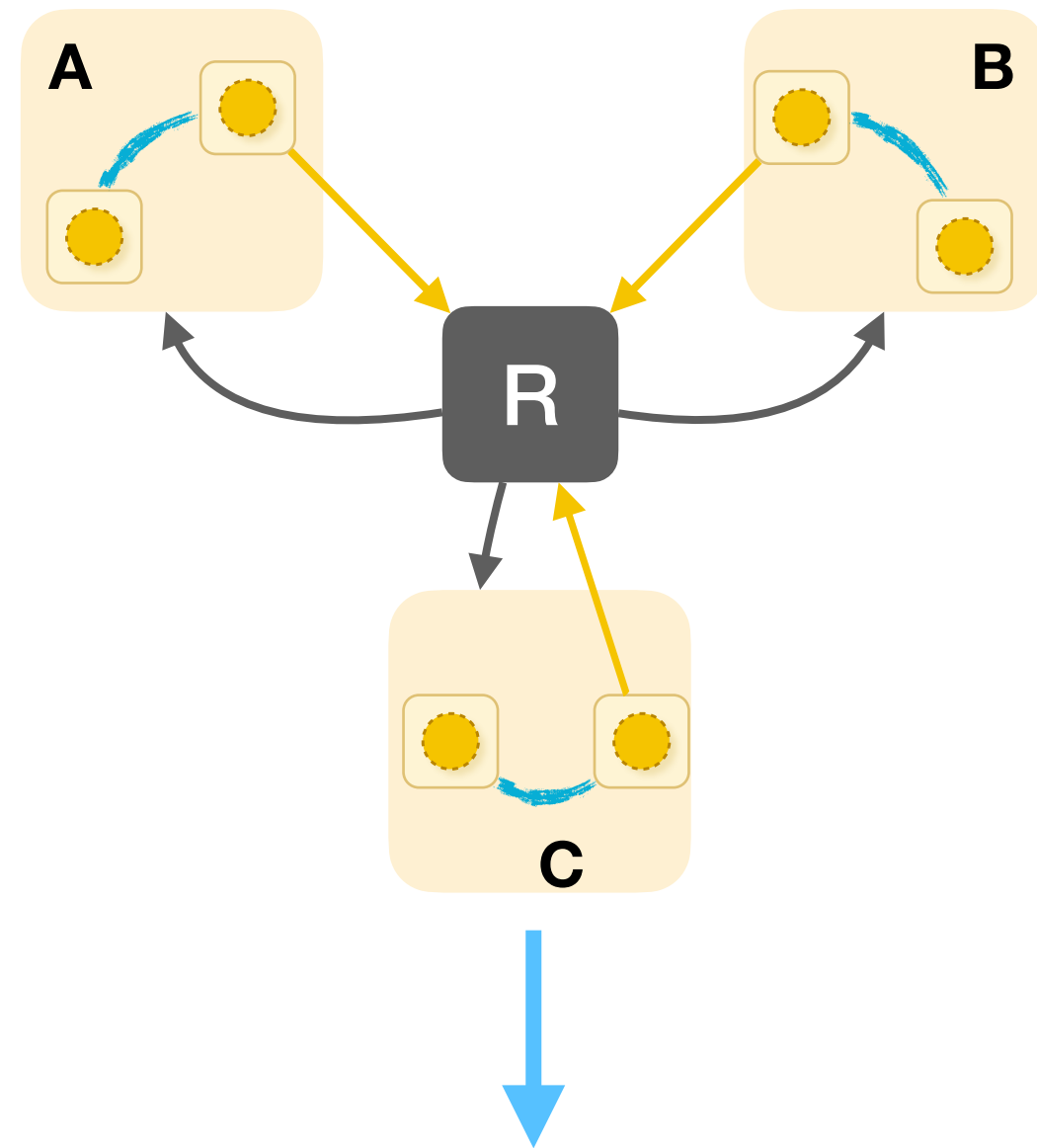
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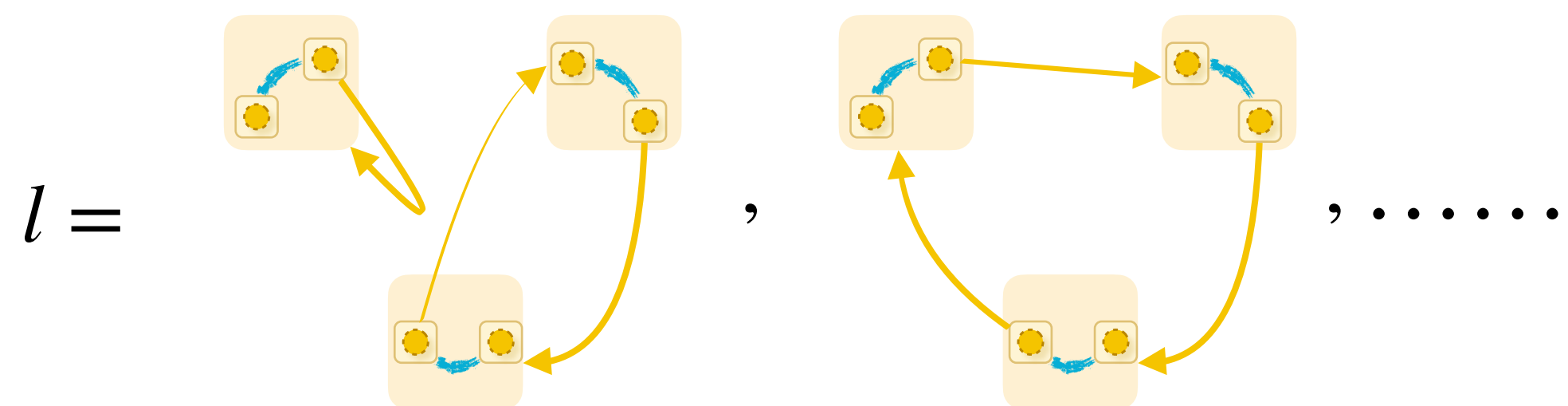
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The Gedankenexperiment

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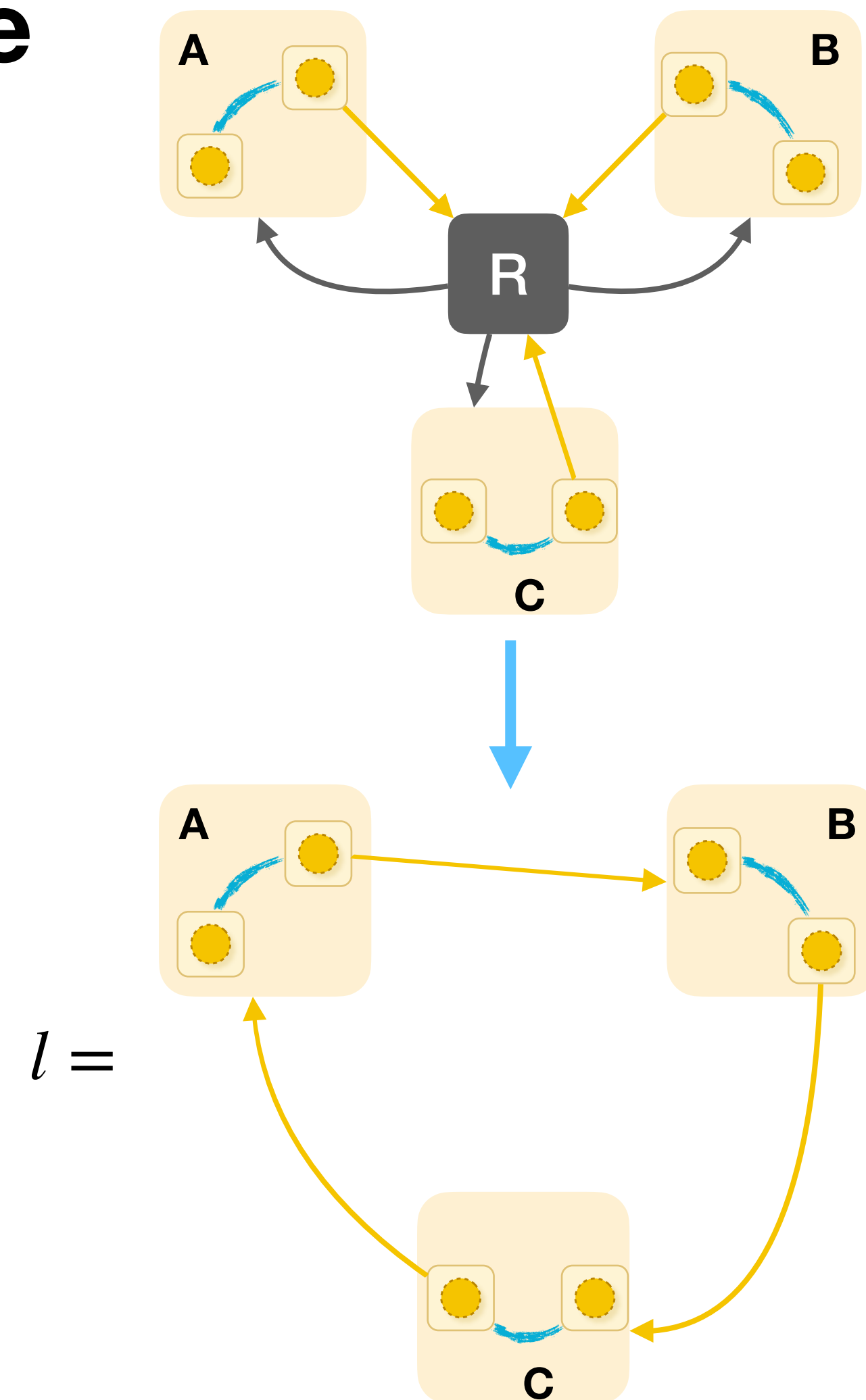


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The Gedankenexperiment

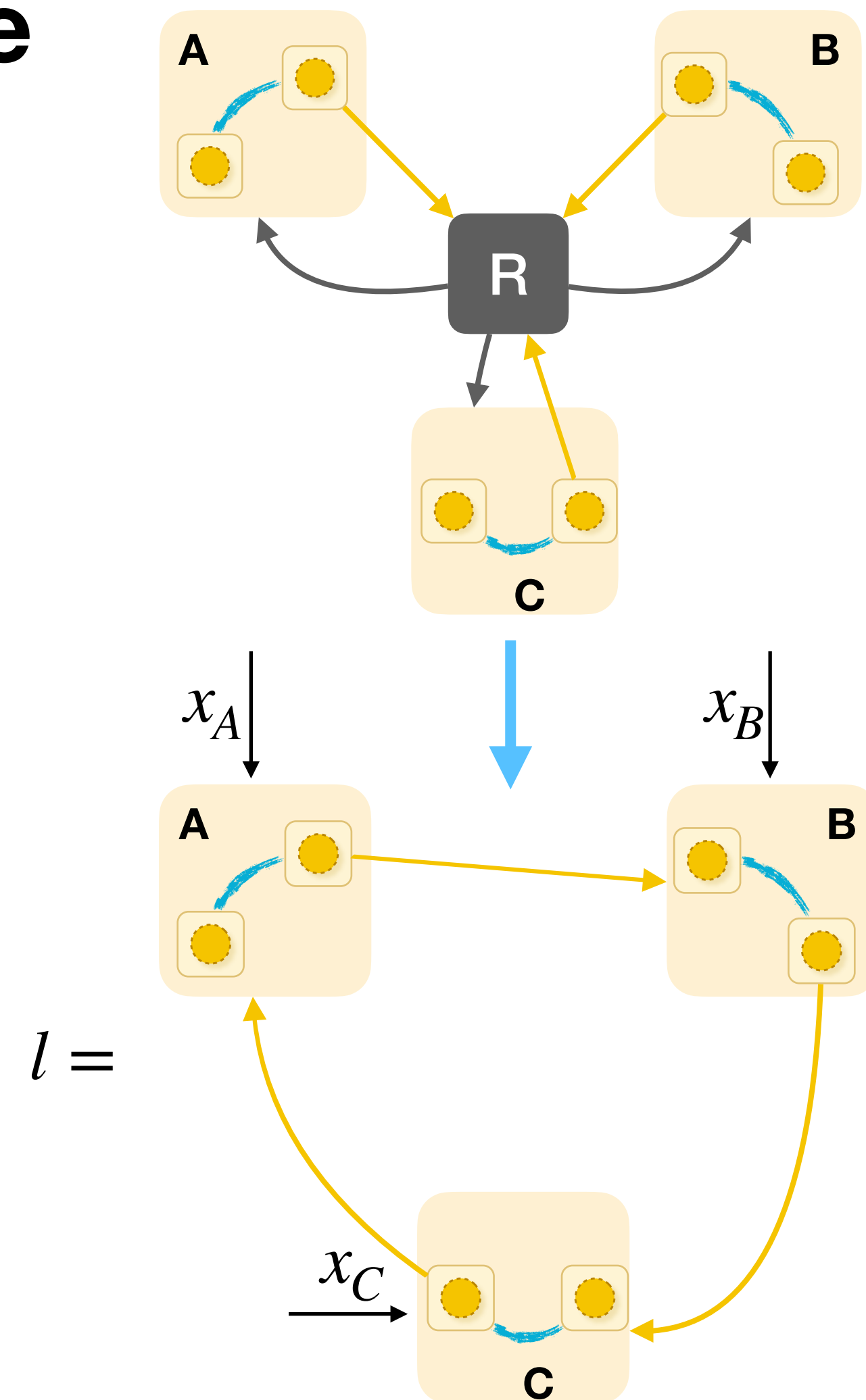
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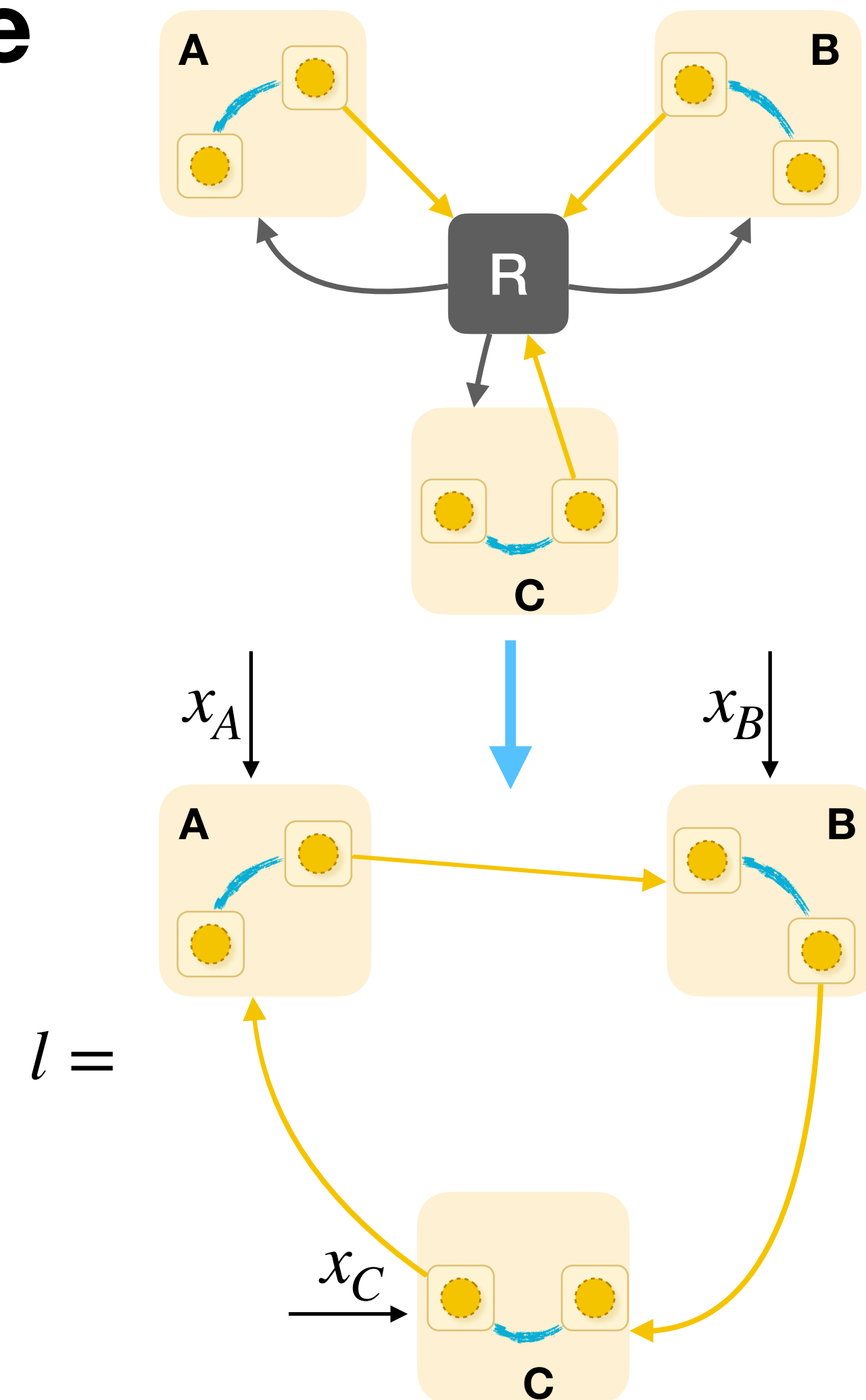
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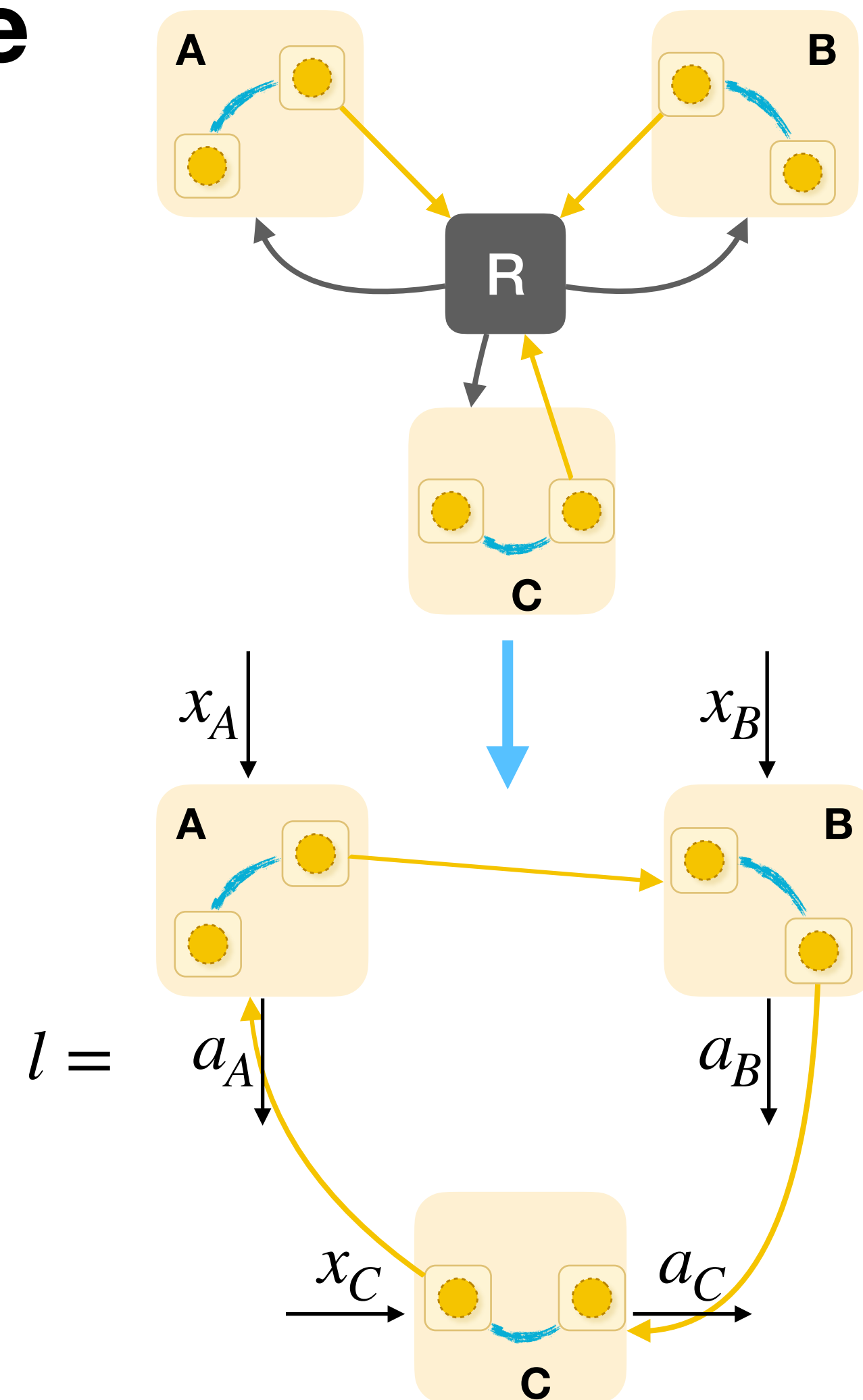
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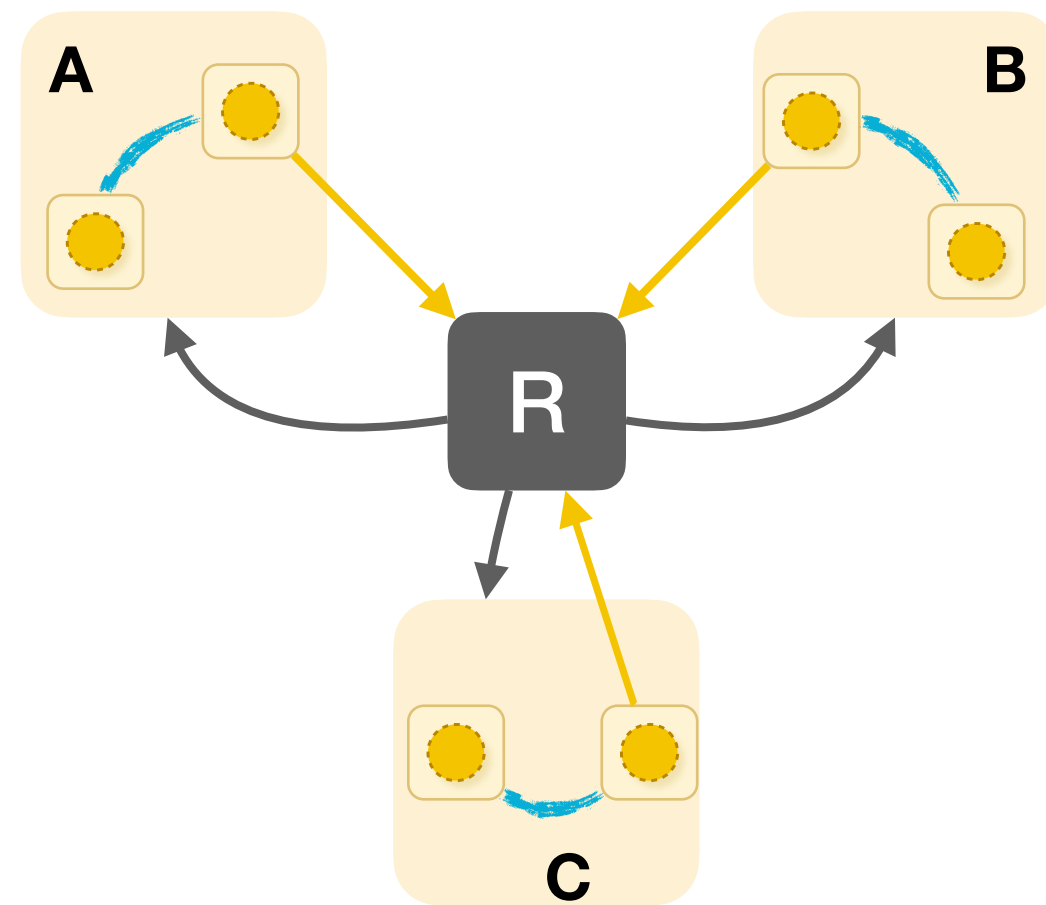


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The Gedankenexperiment

The febit strategy and correlation

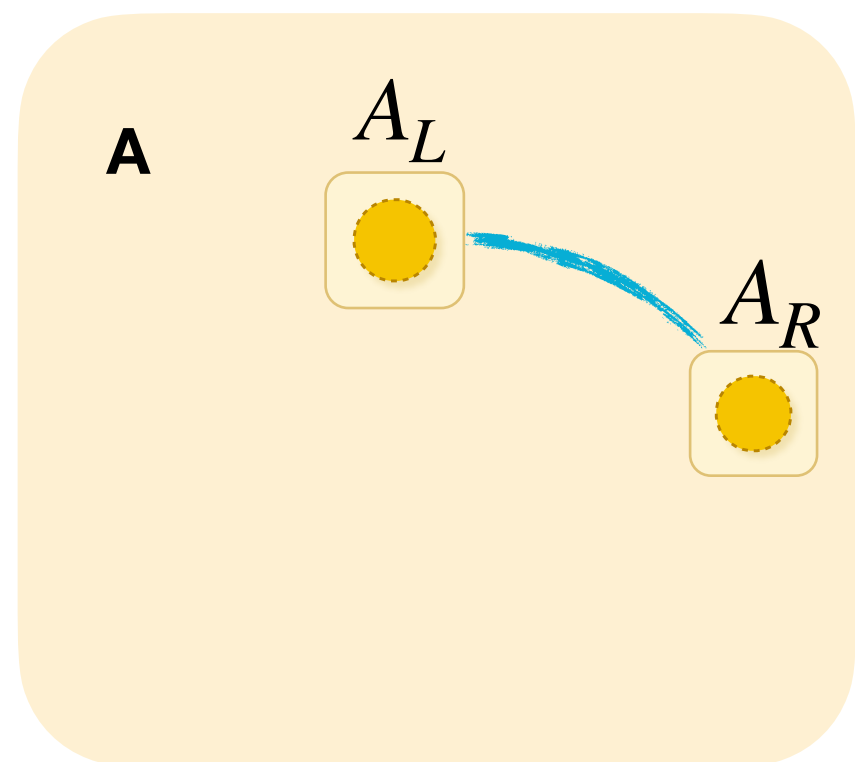
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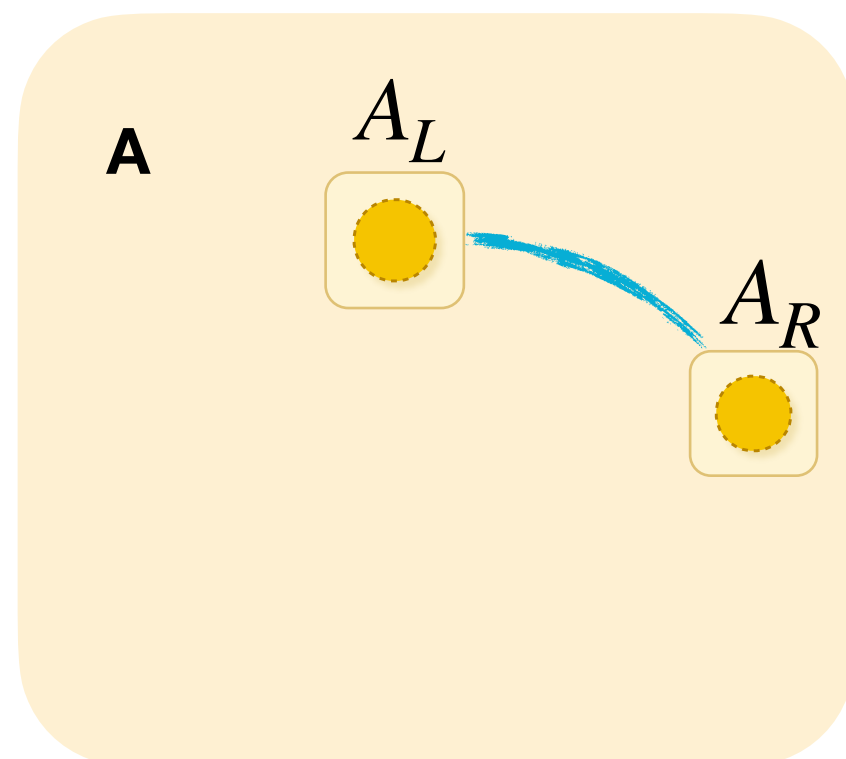


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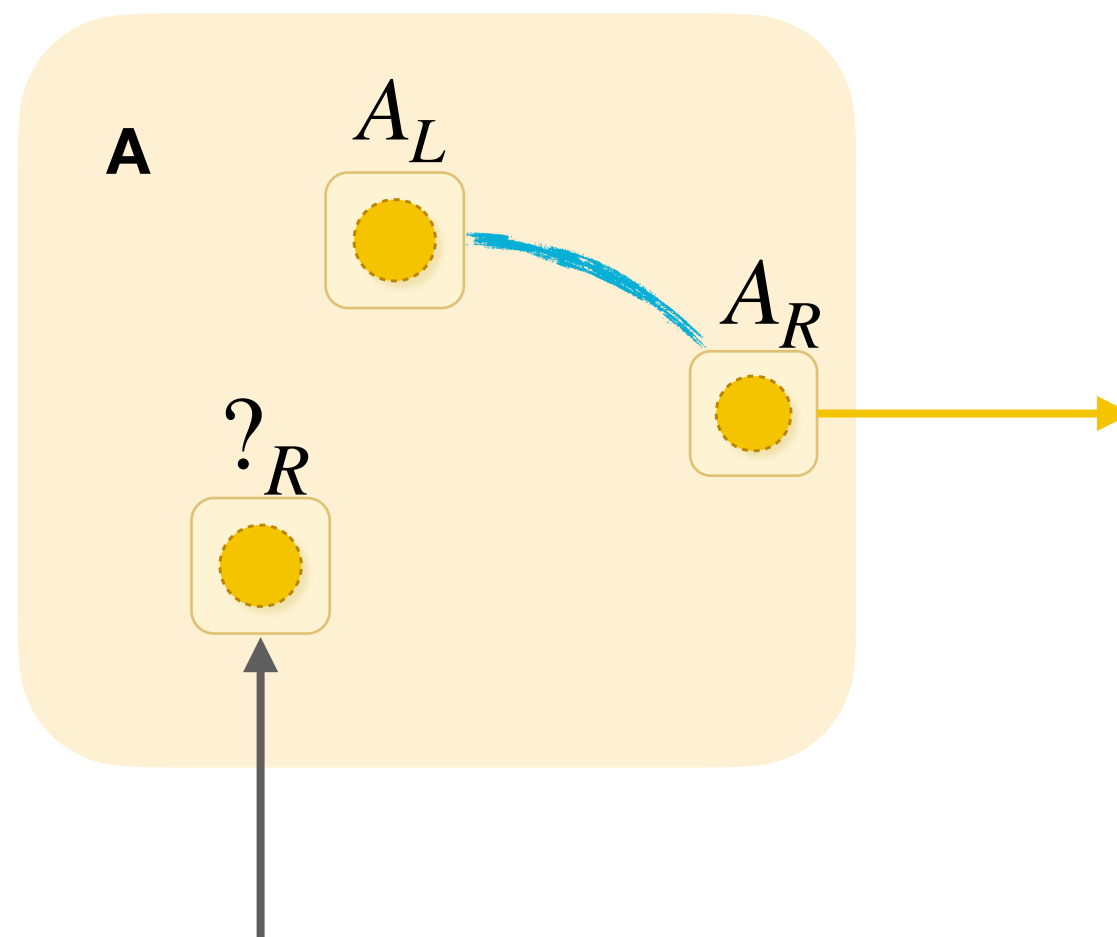


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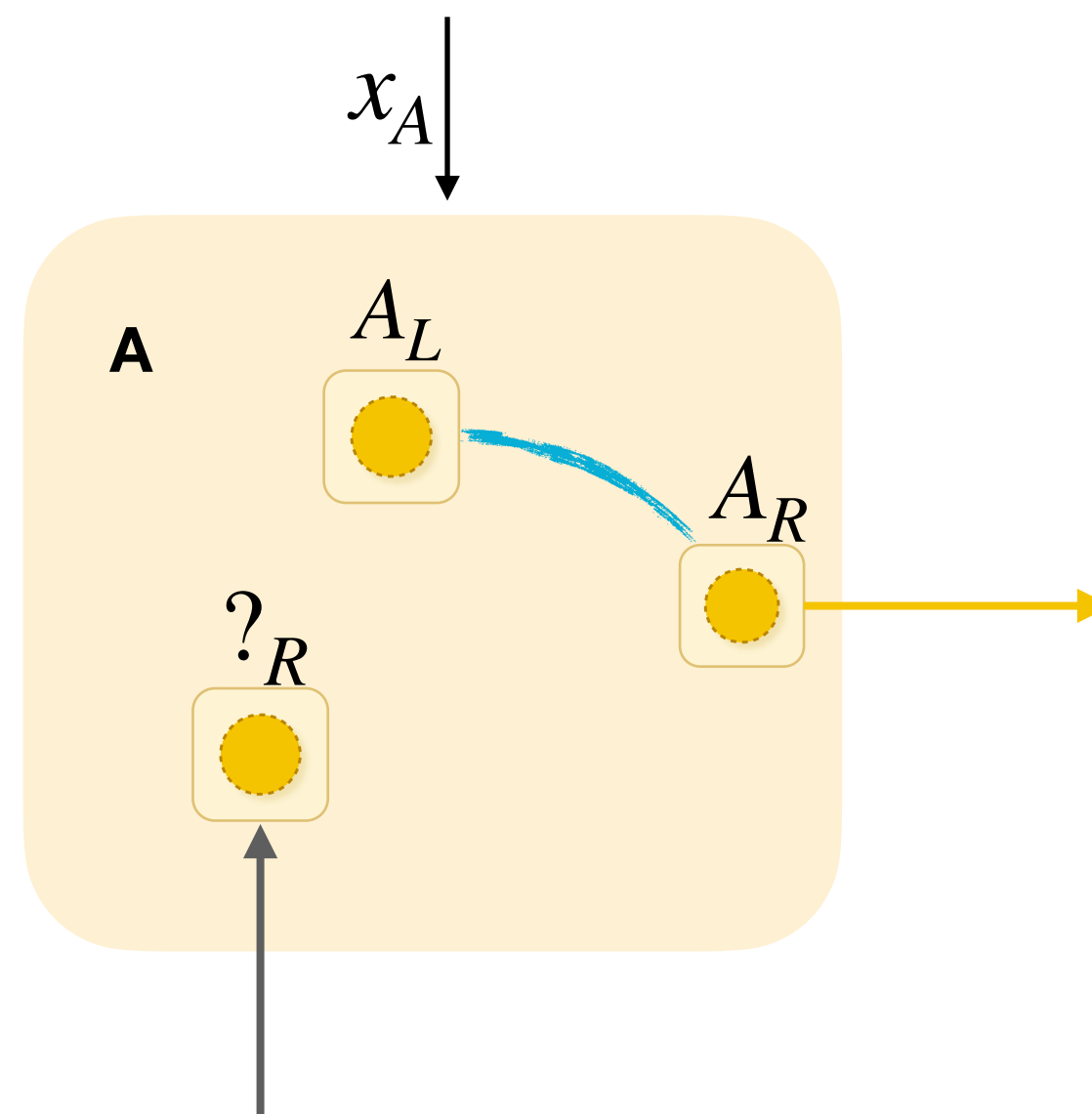


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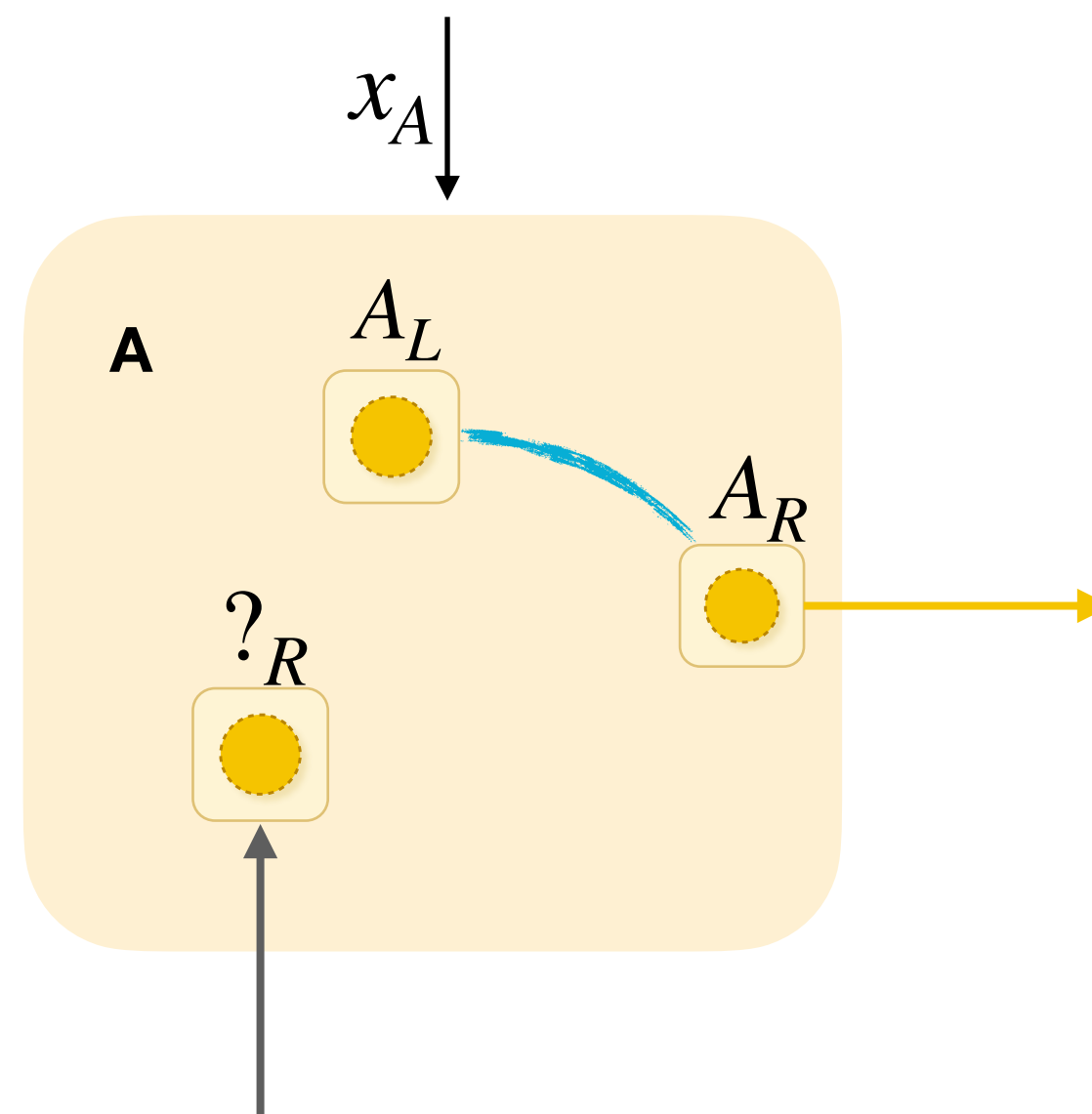
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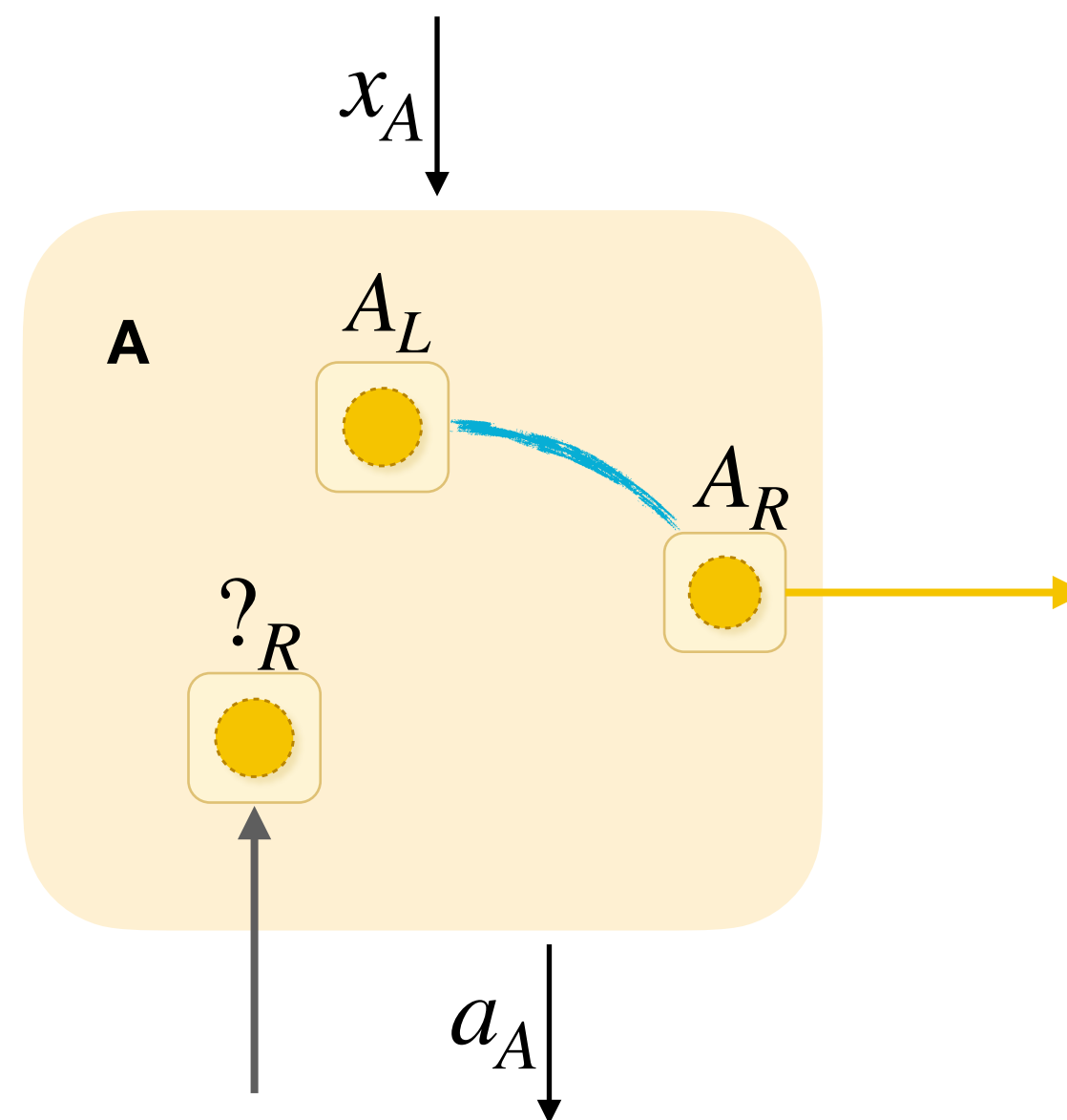
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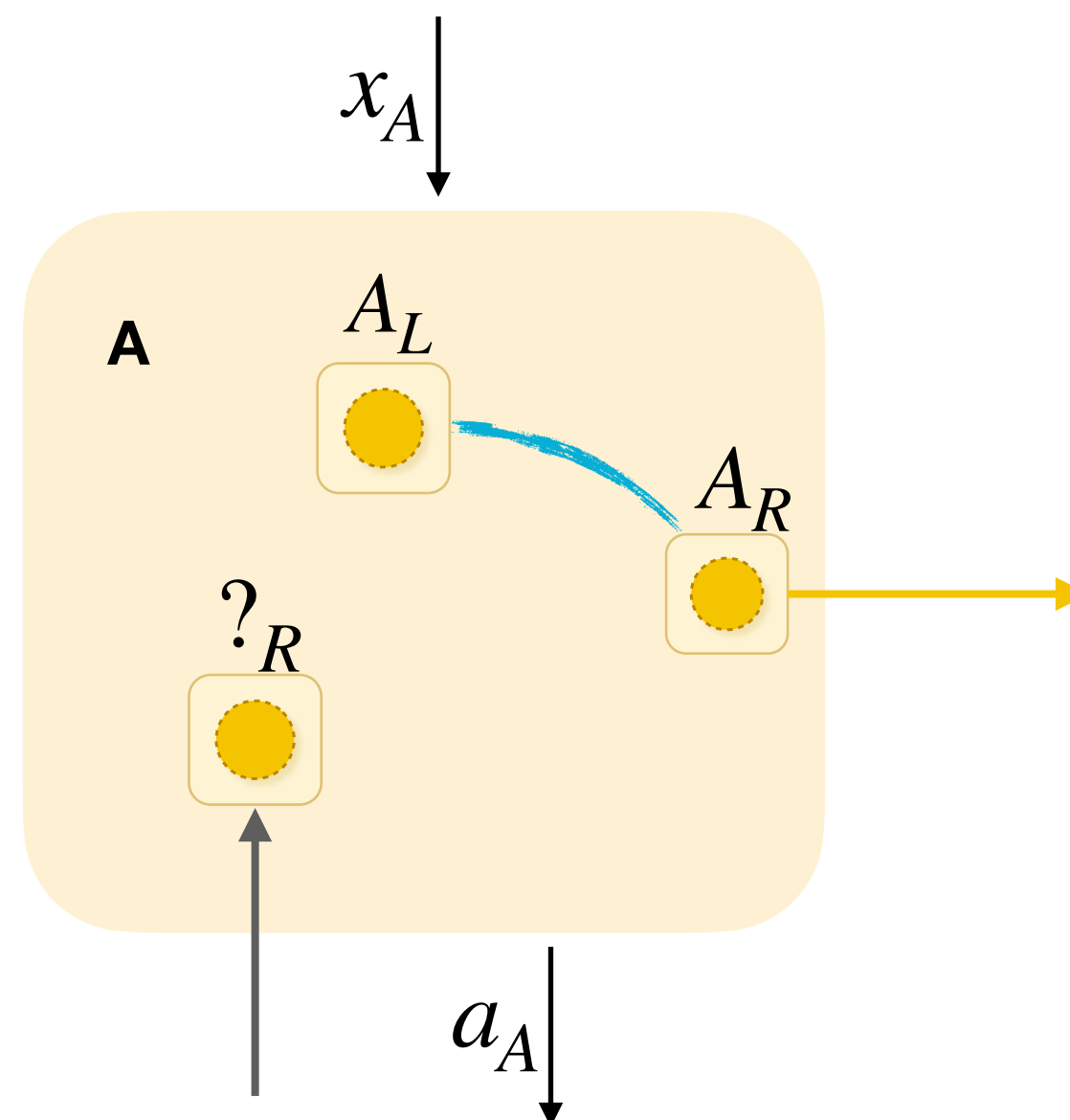


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The Gedankenexperiment

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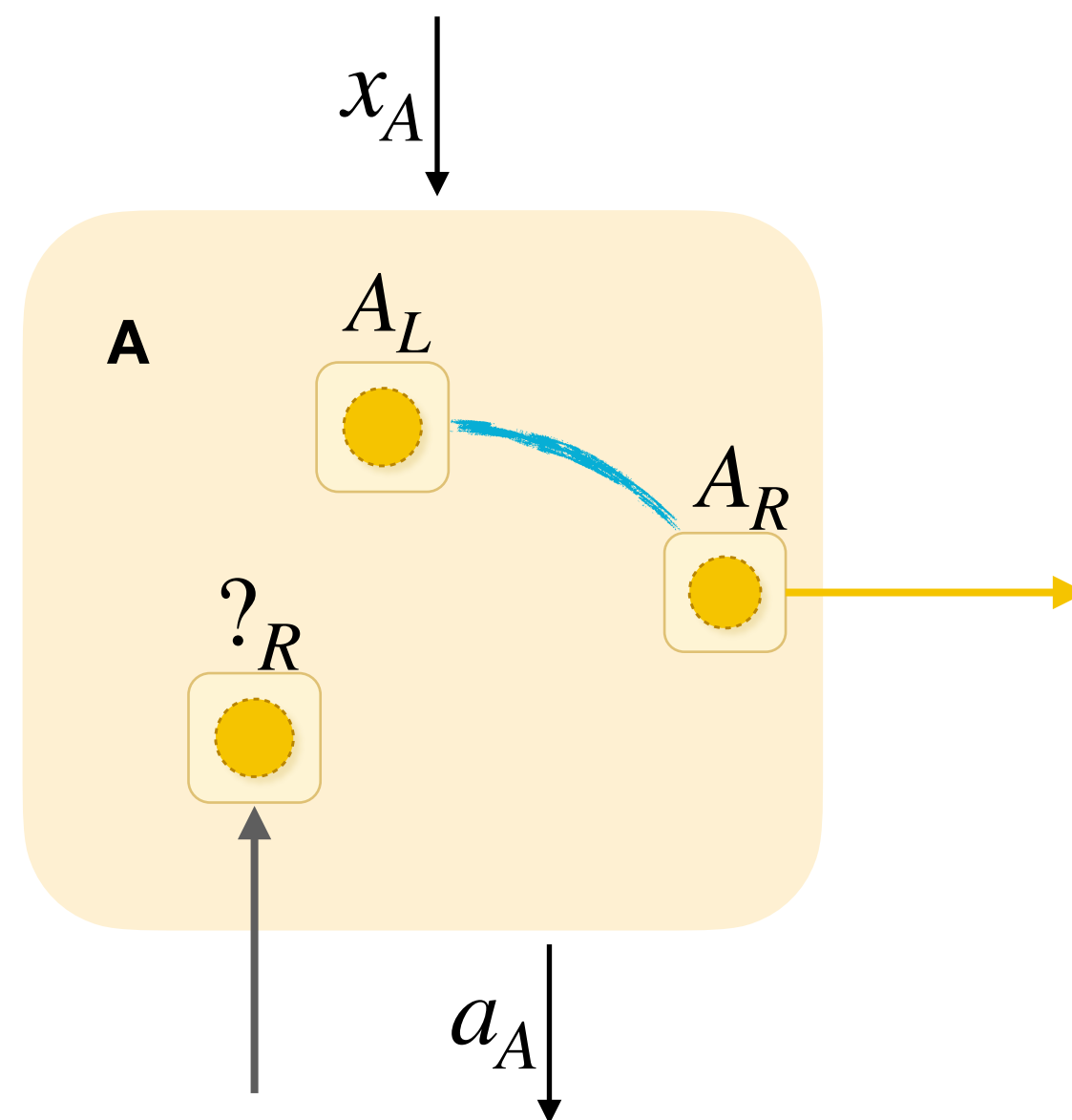


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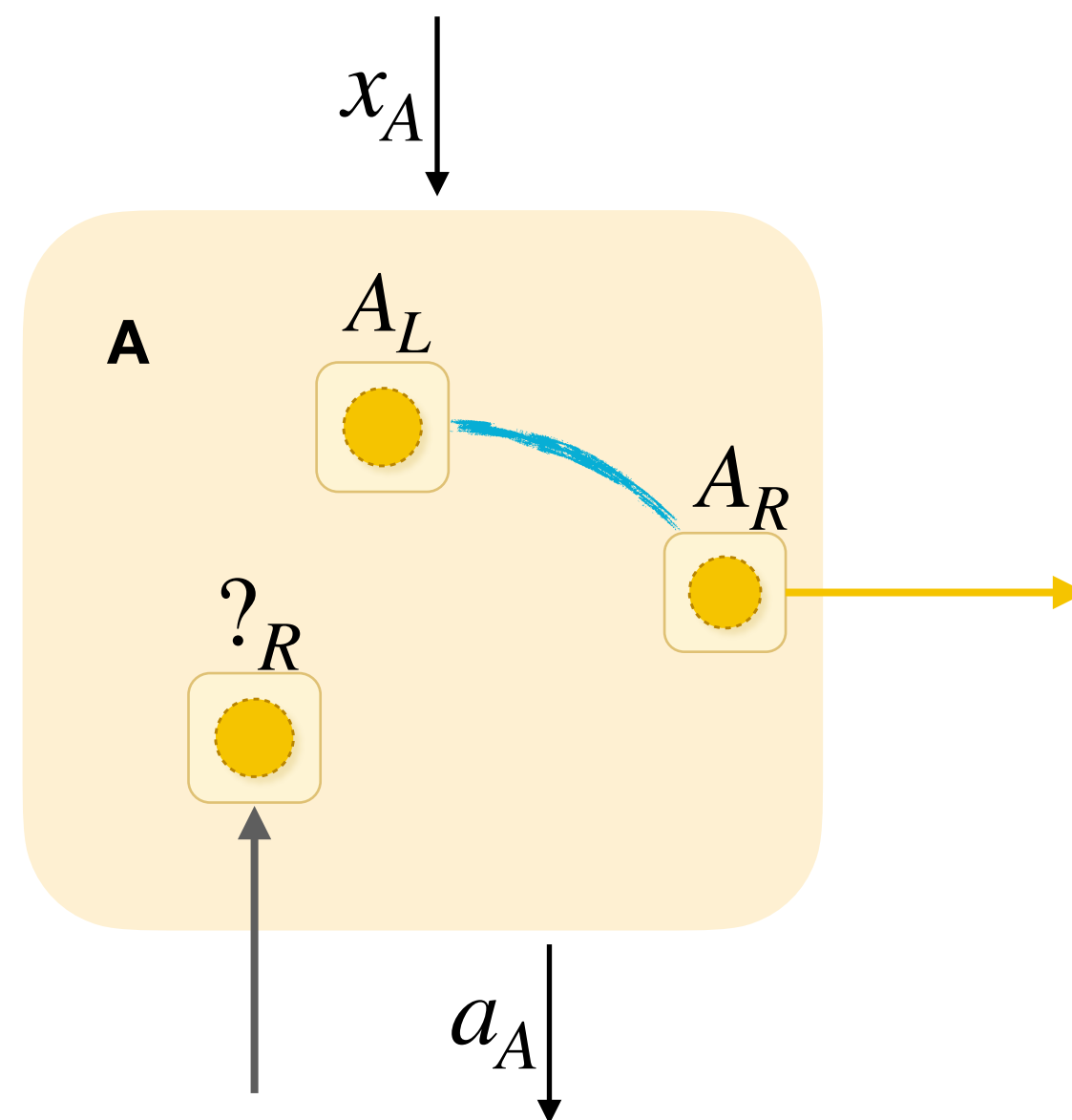


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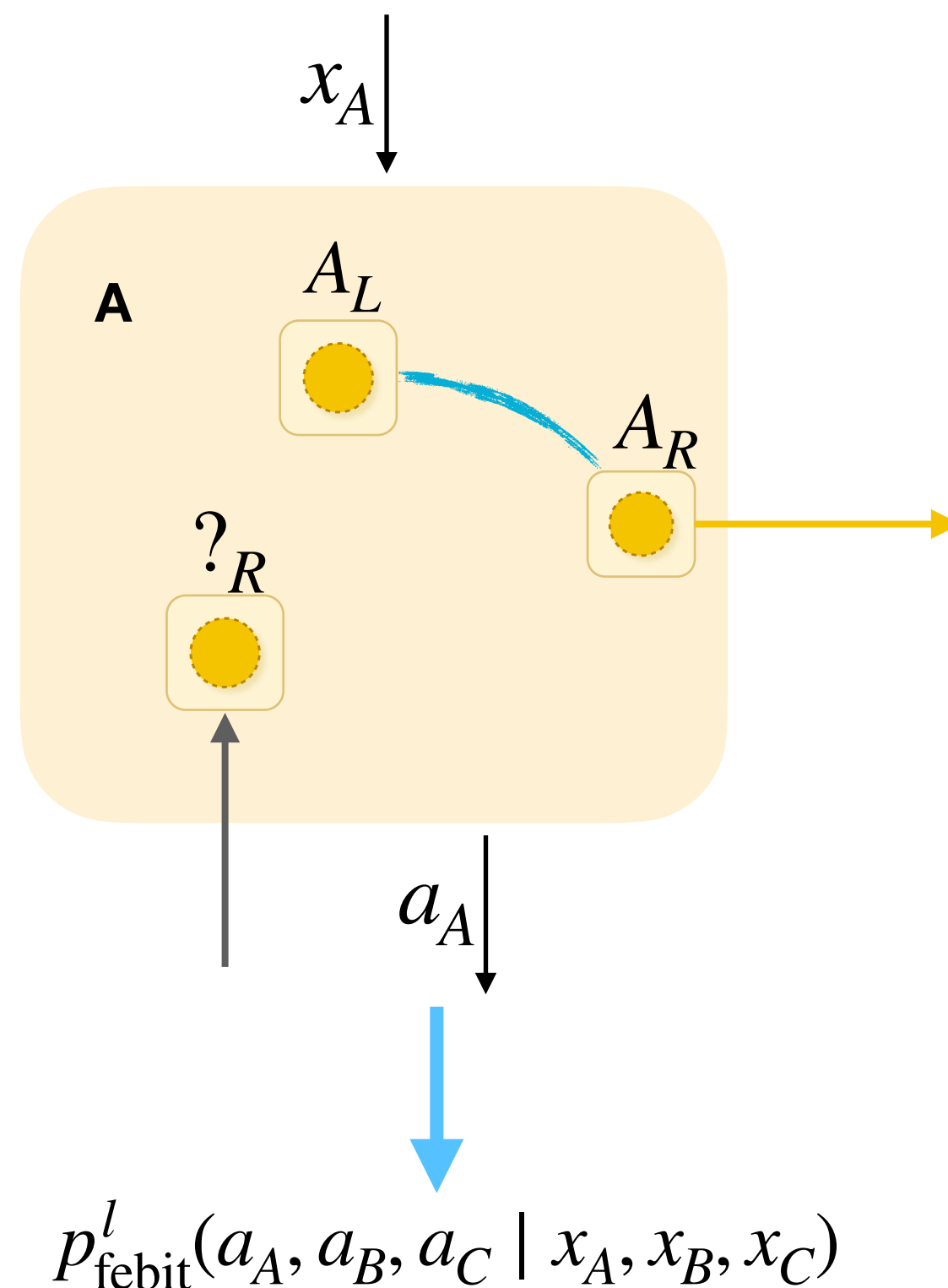
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2-dim state space $\implies$ orthogonal measurements $\Pi_{-}^{x_A}, \Pi_{+}^{x_A}$ for $a_A = 1_-, 1_+$

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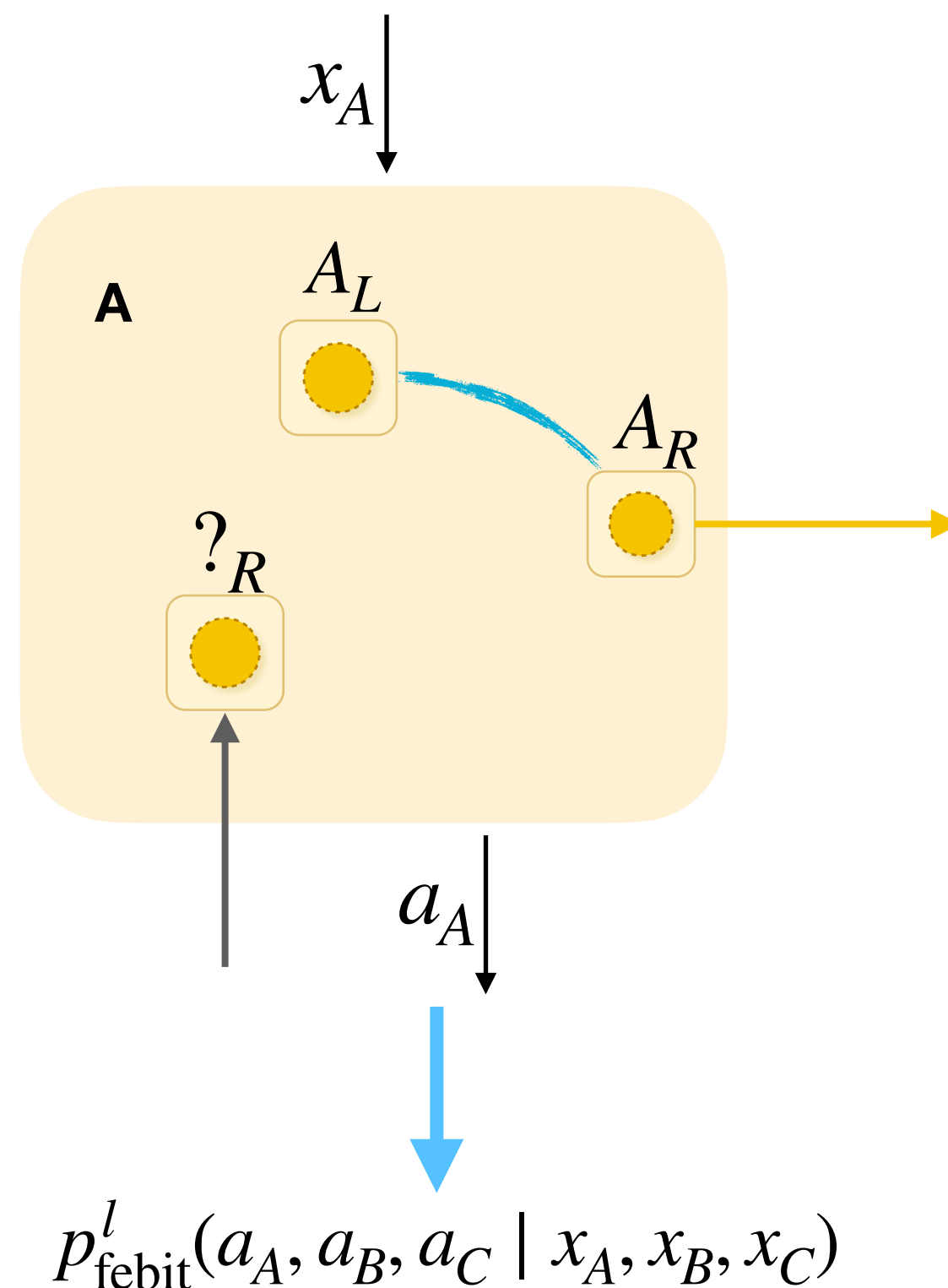
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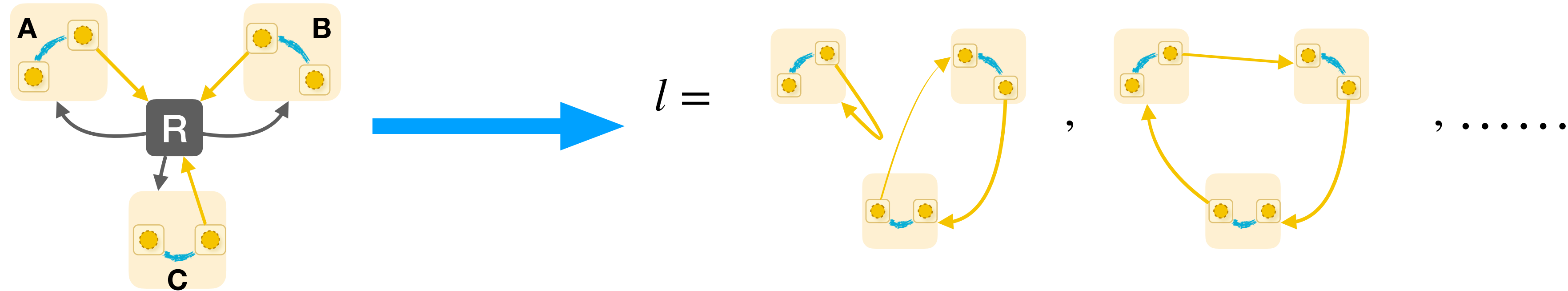
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• $1 : \text{superposition of } | \text{□} \text{●} \rangle_{A_L ?_R} \text{ and } | \text{●} \text{□} \rangle_{A_L ?_R}$

All fermionic operations here are physical w.r.t parity superselection

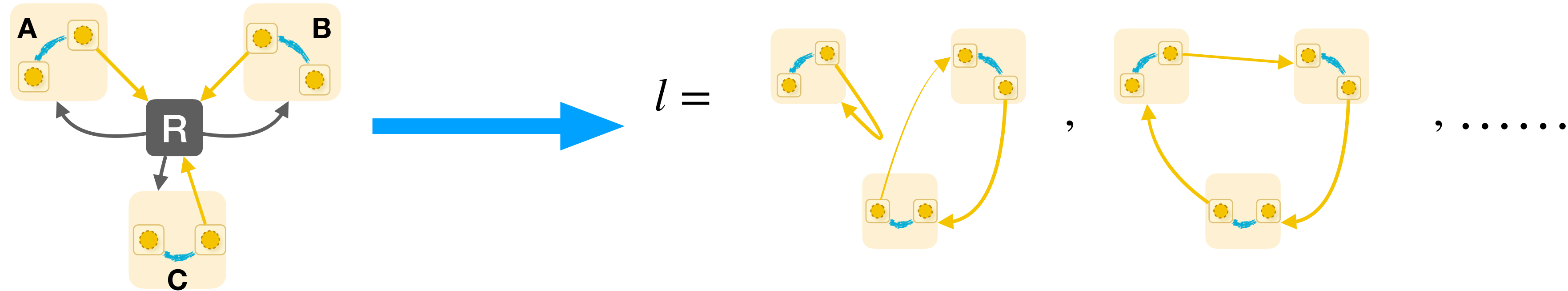
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Main result



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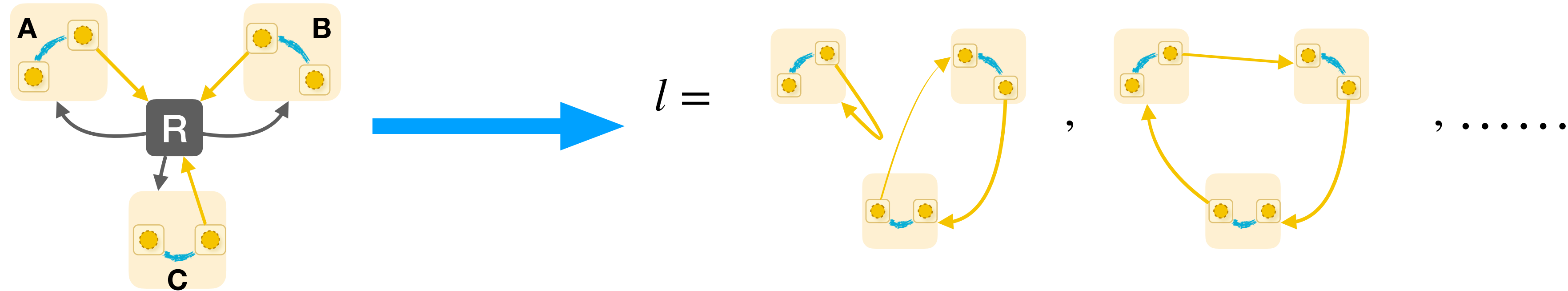
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Main result

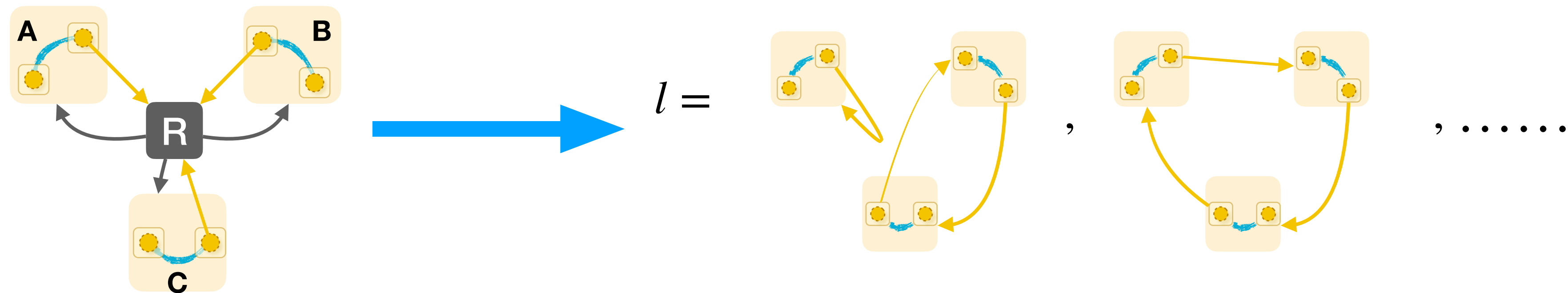


In this distributed network, under the same communication rule, no qubit-based strategy can reproduce $p_{\text{febit}}^l(a_A, a_B, a_C \mid x_A, x_B, x_C)$ for all configurations l simultaneously!

1. **A, B, C** each locally prepare a bipartite system, keep one and send one to **R**.
2. **R** secretly choose a configuration l and redistributes.
3. **R** sends each **A, B, C** a bit $x_i \in \{0, 1\}$
4. With local operations, **A, B, C** outputs $a_i \in \{0, 1_-, 1_+, 2\}$
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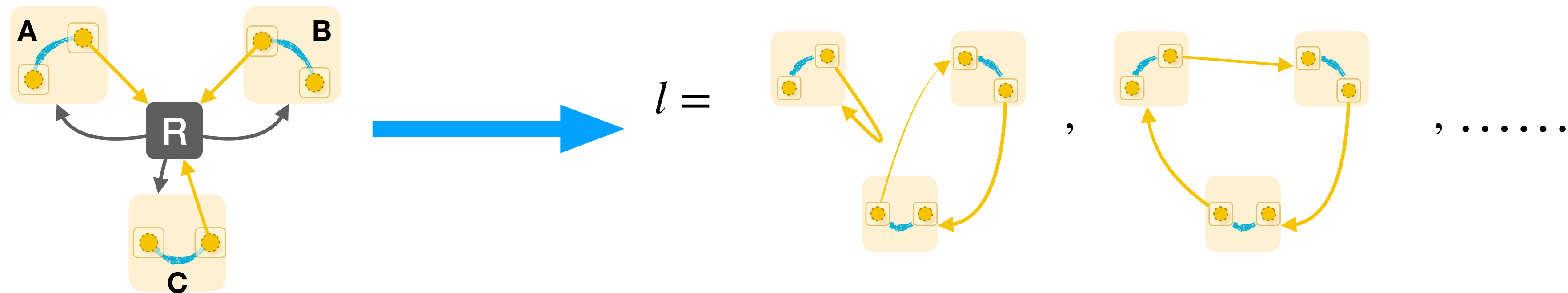
Local vs global entanglement



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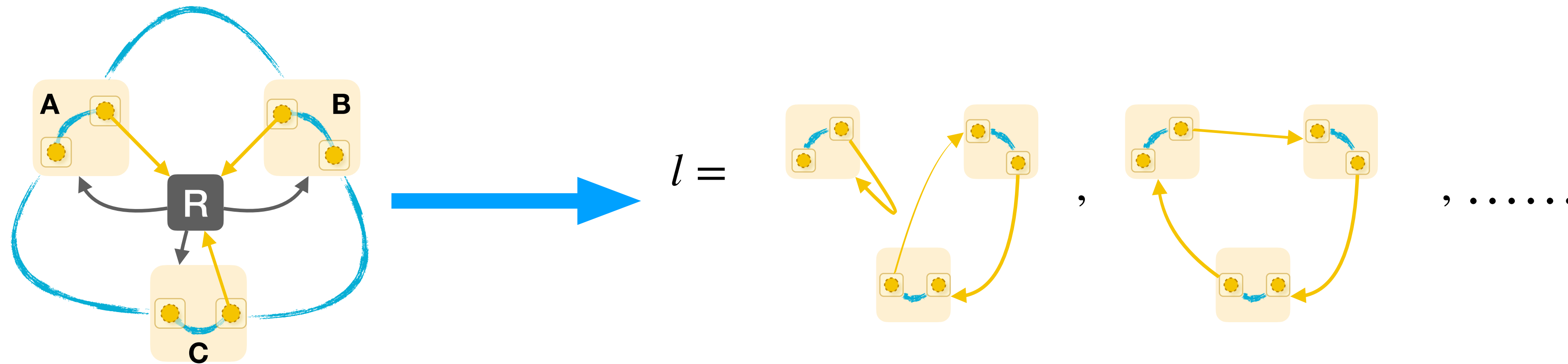


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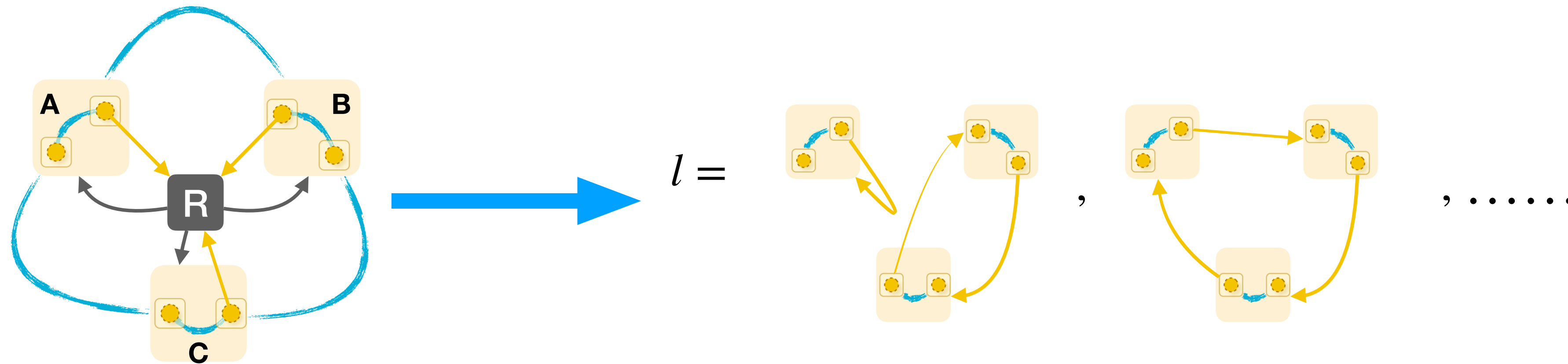


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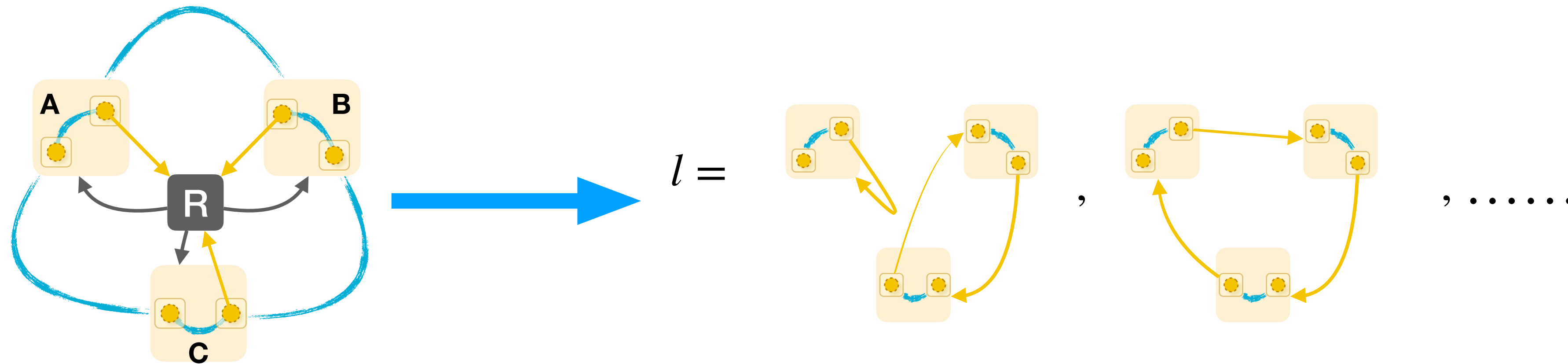


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- Get global entanglement needs more communication steps

Local vs global entanglement



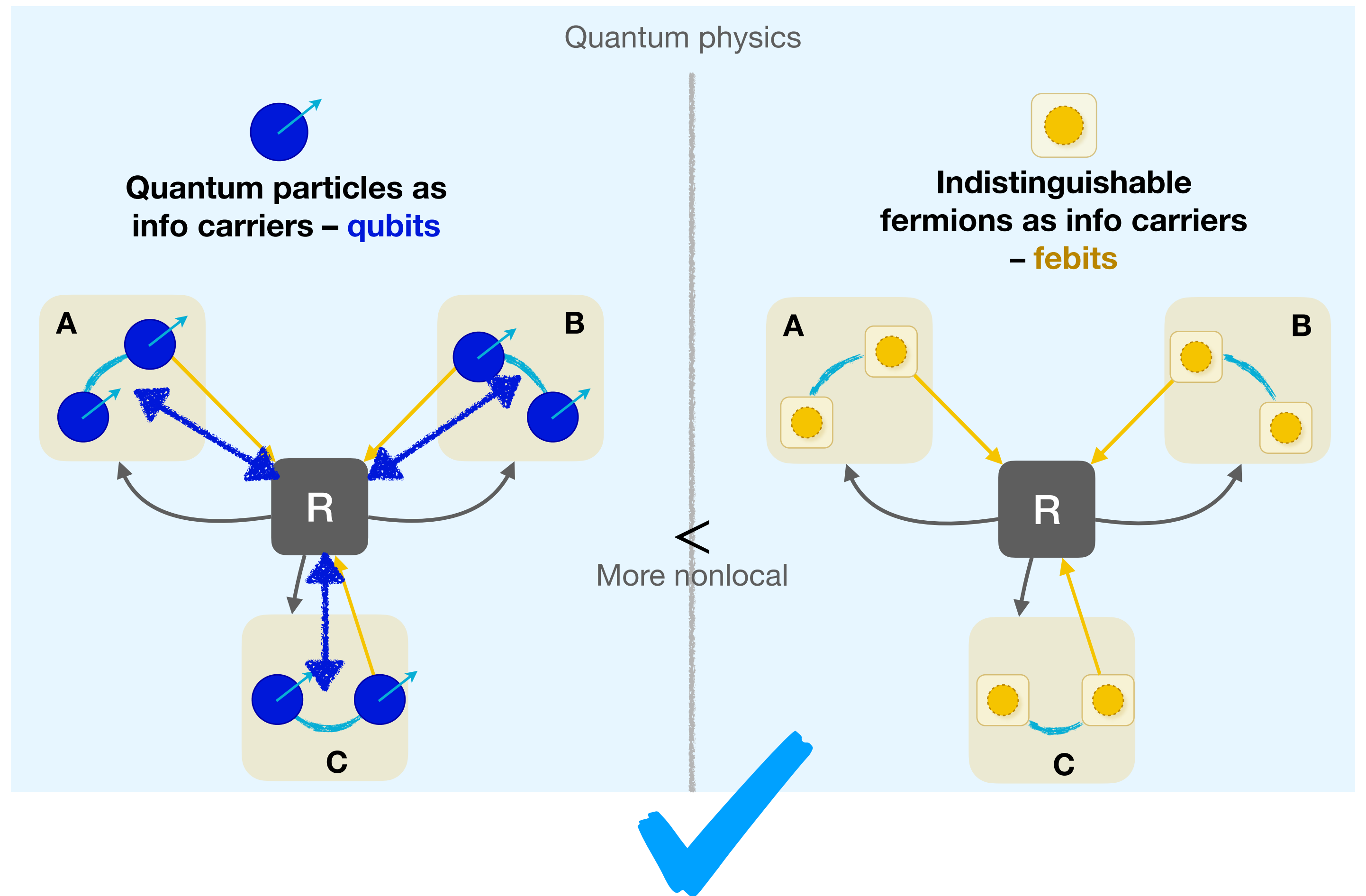
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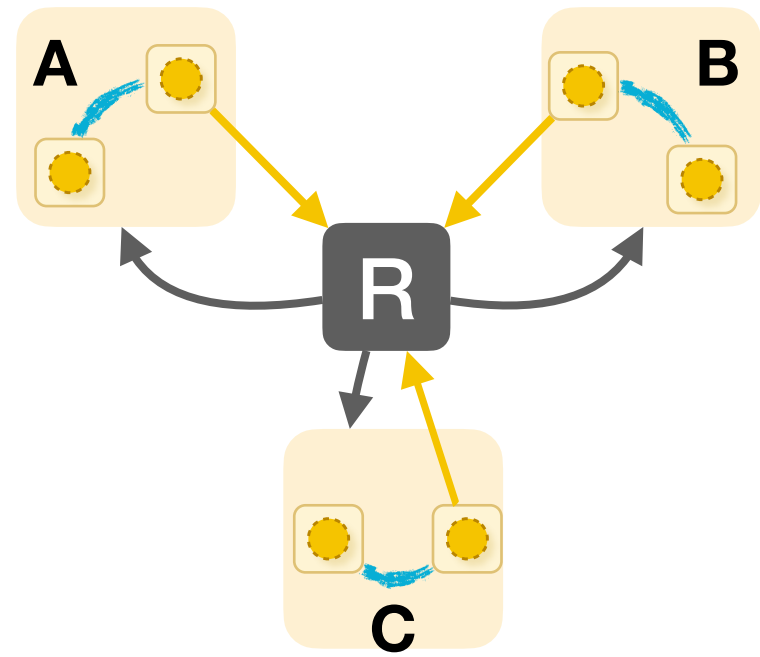
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- Get global entanglement needs more communication steps
- ~Nonlocality without pre-shared entanglement

Outline

- What are febits
- Intuition for febit advantage
 1. Detecting exchange phase
 2. Distributed quantum network
- The Gedankenexperiment
 - Proof: self-testing of qubits
 - Conclusions



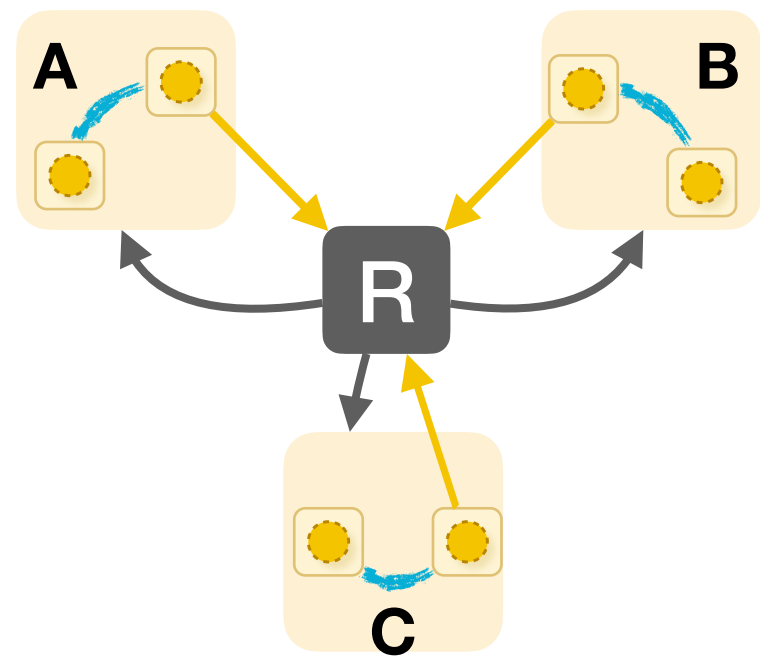
Proof: Simple qubit strategy for fixed l



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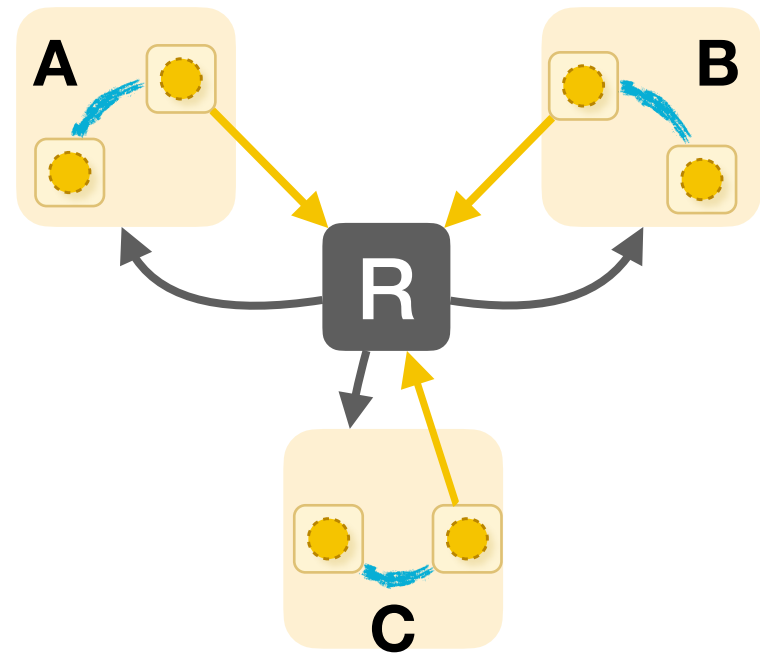


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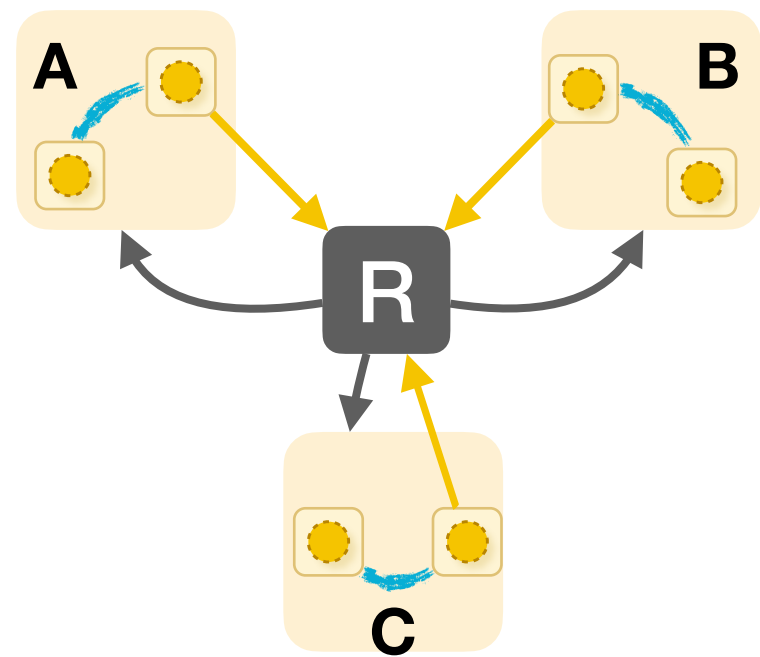


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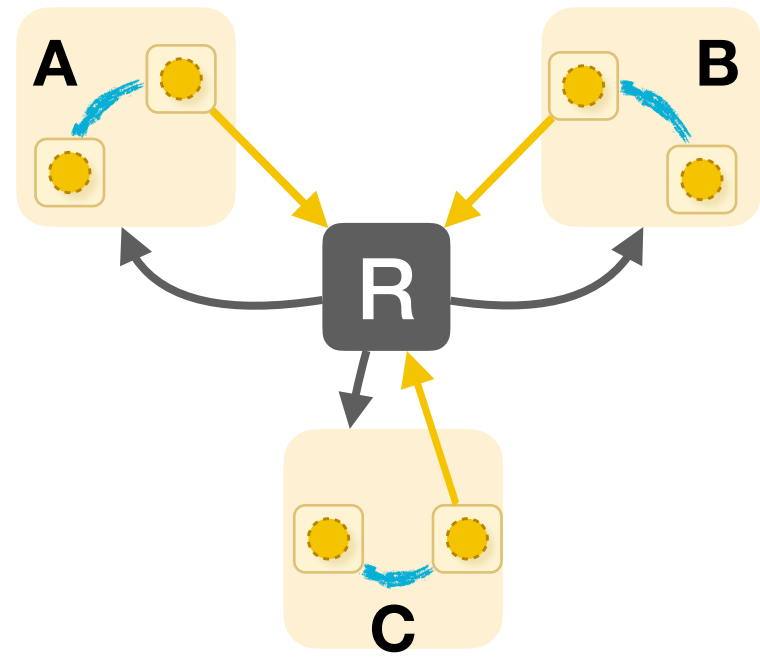


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GHZ state with 01 = $\uparrow$, 10 = $\downarrow$!
- Want a qubit strategy to have a global state $(|01,01,\dots\rangle + (-1)^{N-1}|10,10,\dots\rangle)/\sqrt{2}$ when count 1 qubit everywhere

Proof: Simple qubit strategy for fixed l

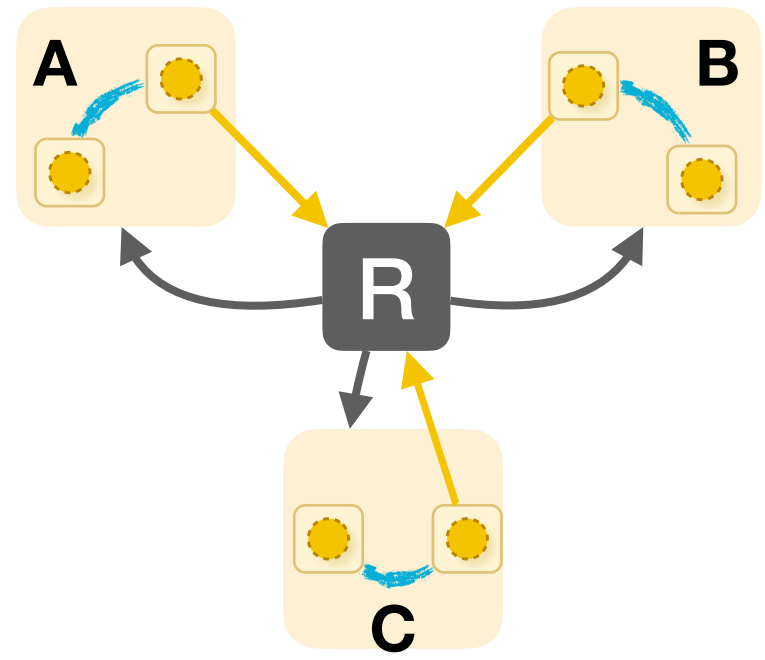


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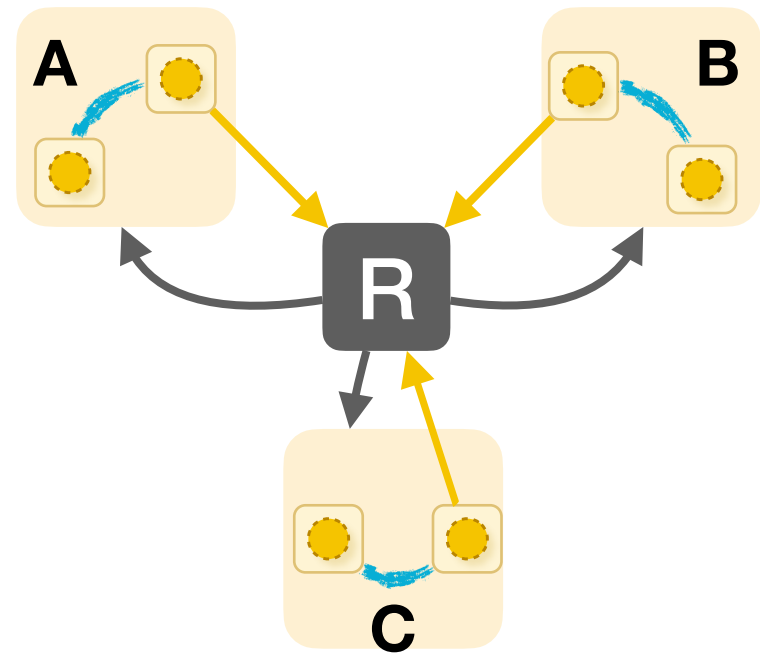


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- Loop l_N of size N , count 1, febit strategy gives a global state: $(|01,01\dots\rangle + (-1)^{N-1}|10,10\dots\rangle)/\sqrt{2}$
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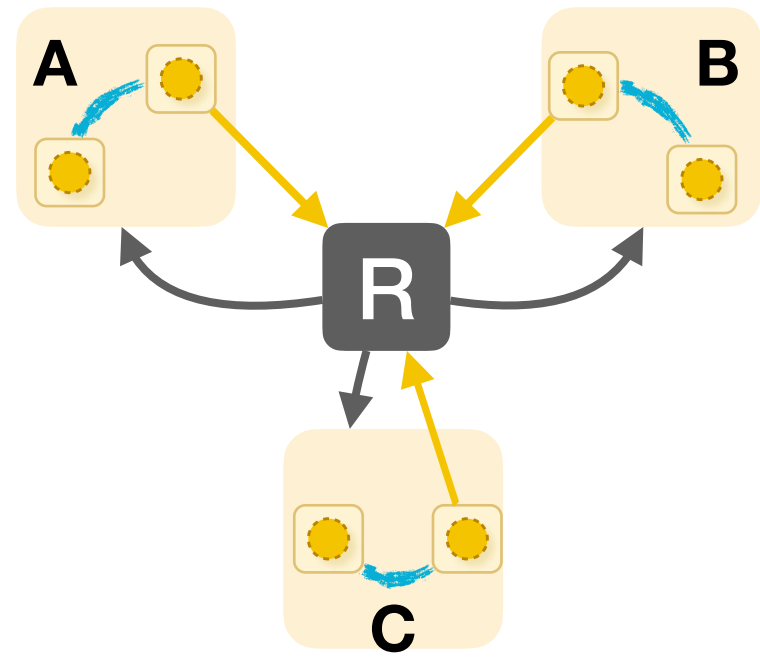


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- Each party j can locally prepare $(|01\rangle + e^{i\phi_j}|10\rangle)/\sqrt{2}$, s.t. $\prod_j e^{i\phi_j} = (-1)^{N-1}$
- With appropriate measurement operators we get $p_{\text{febit}}^{l_N}$

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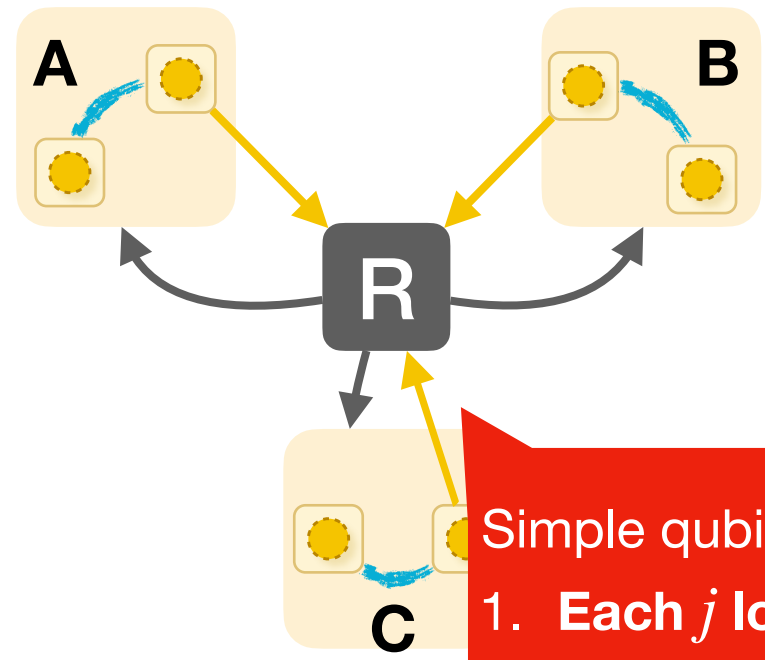
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- With appropriate measurement operators we

In our game, j does not know l , so they cannot choose the local phase to that simultaneously satisfies $p_{\text{febit}}^{l_N}$ and $p_{\text{febit}}^{l_{N-1}!}$

Proof: self-testing of all qubit strategies



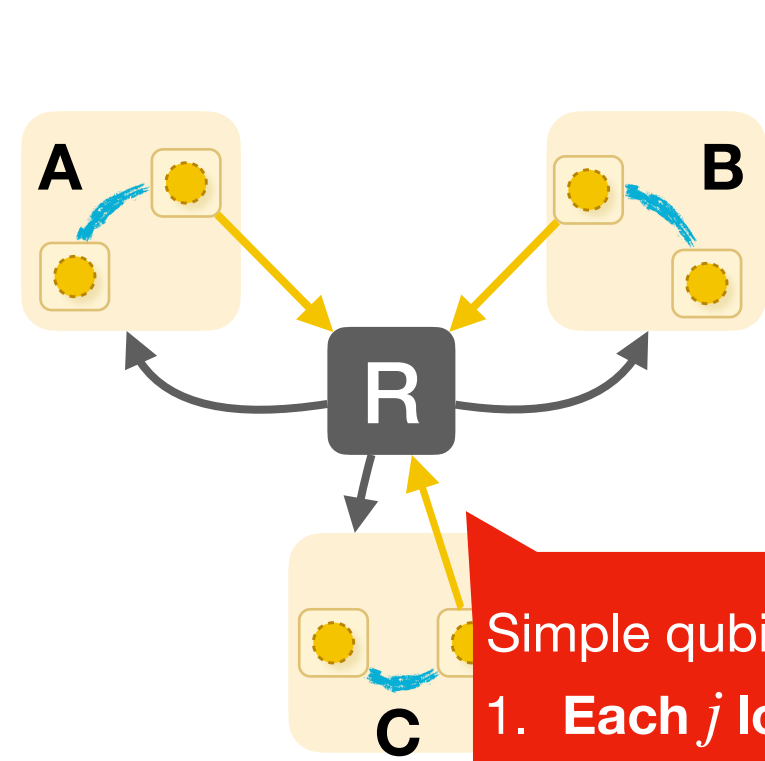
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Simple qubit strategy or loop l of size N :

1. Each j locally prepares $(|01\rangle + e^{i\phi_j}|10\rangle)/\sqrt{2}$, s.t. $\prod_j e^{i\phi_j} = (-1)^{N-1}$
2.
3. Get p_{febit}^l with qubit strategy

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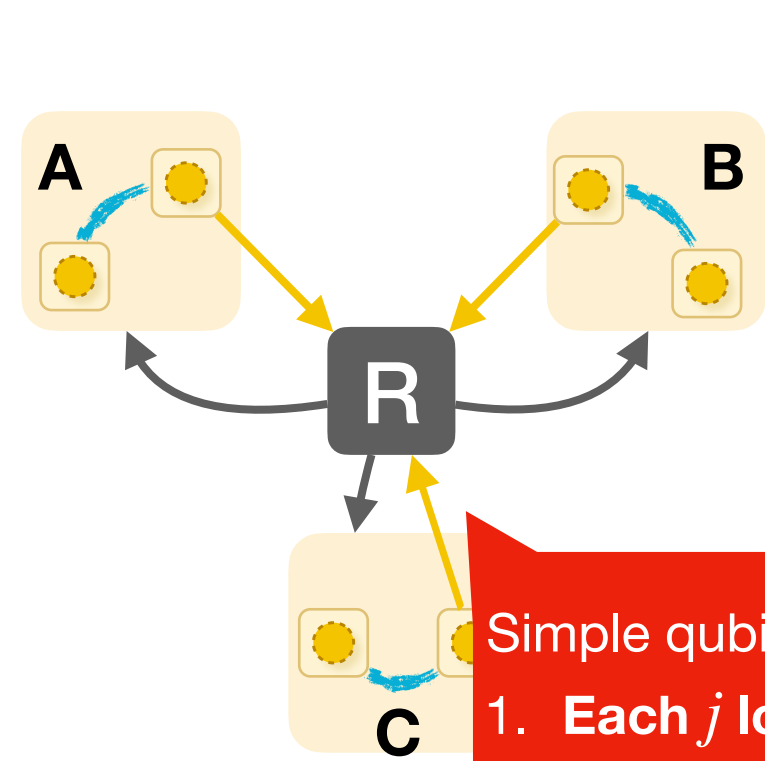
Simple qubit strategy or loop l of size N :

1. Each j locally prepares $(|01\rangle + e^{i\phi_j}|10\rangle)/\sqrt{2}$, s.t. $\prod_j e^{i\phi_j} = (-1)^{N-1}$
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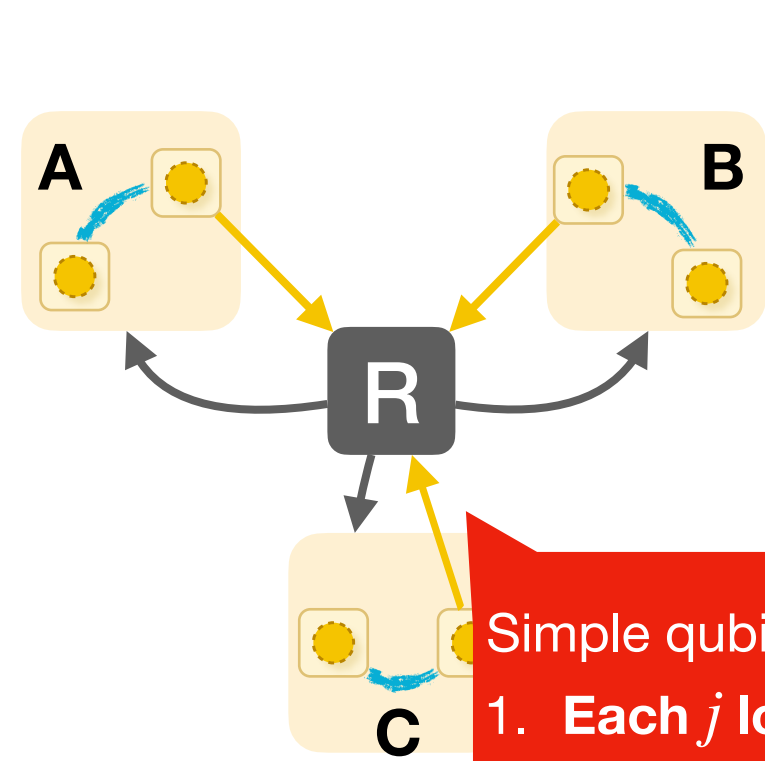
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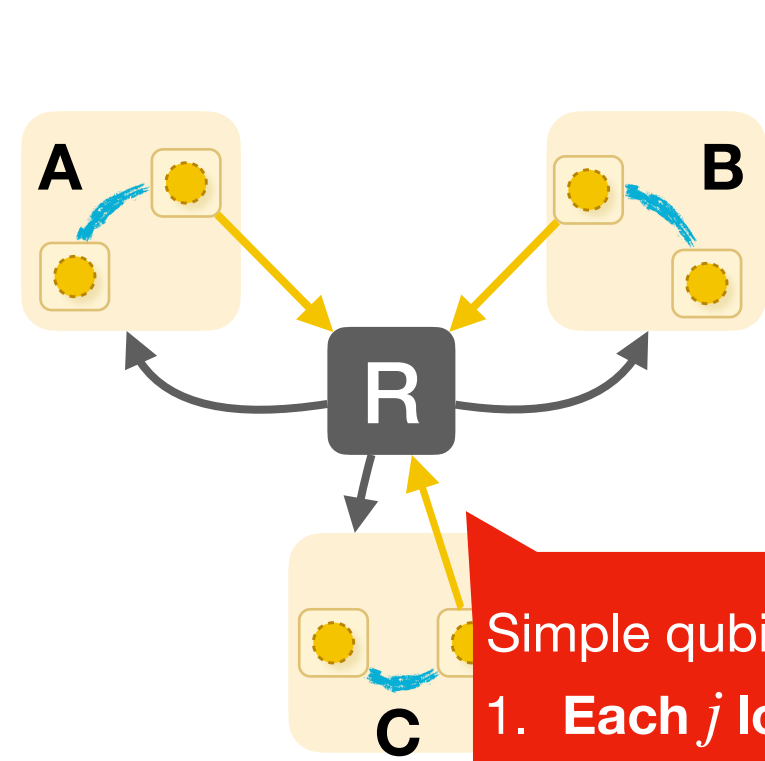
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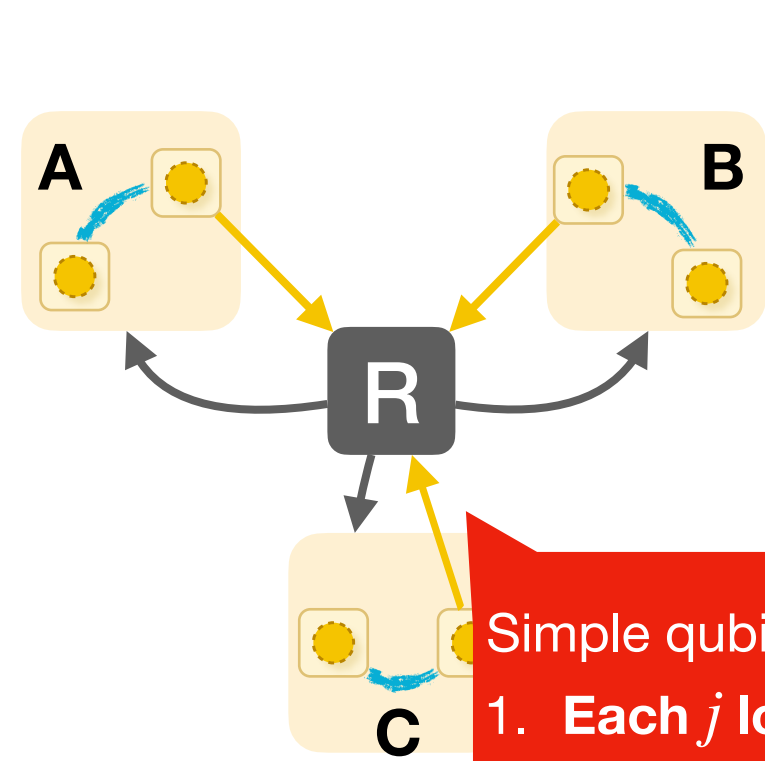
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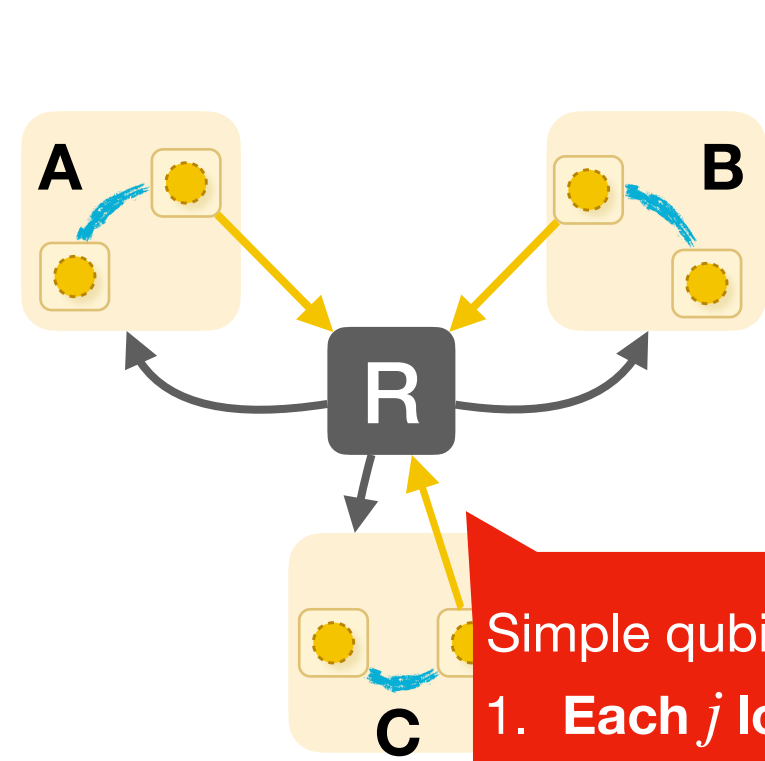
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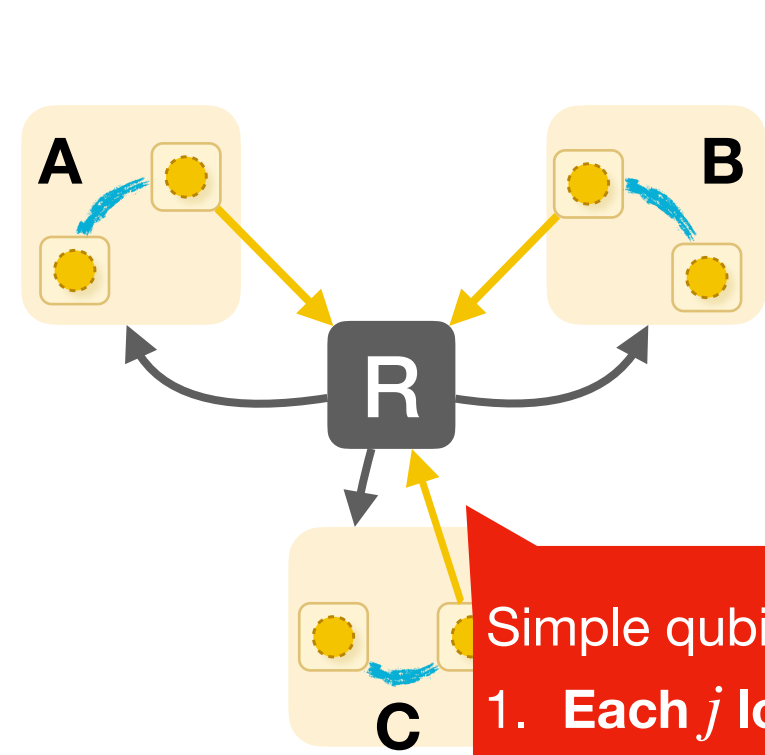
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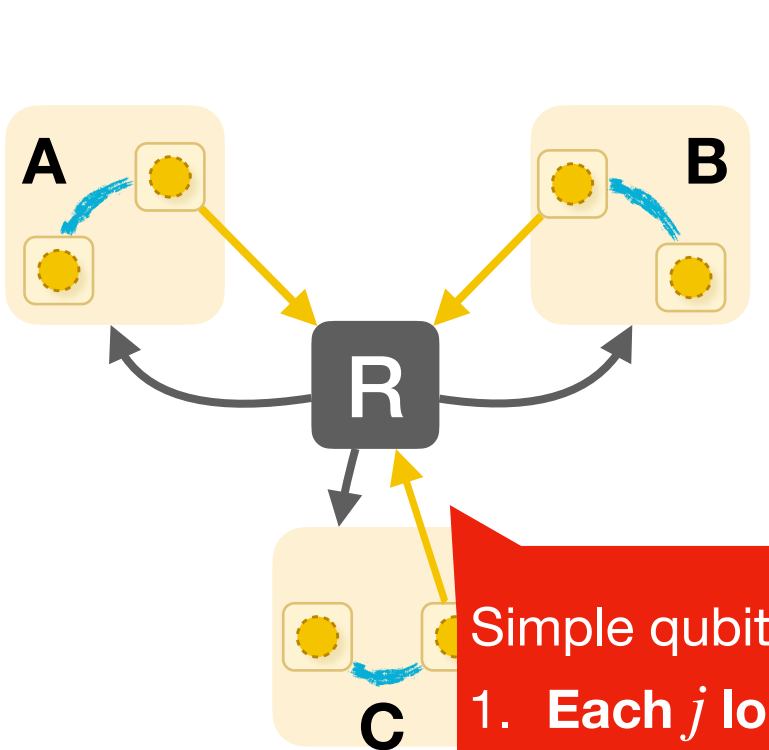
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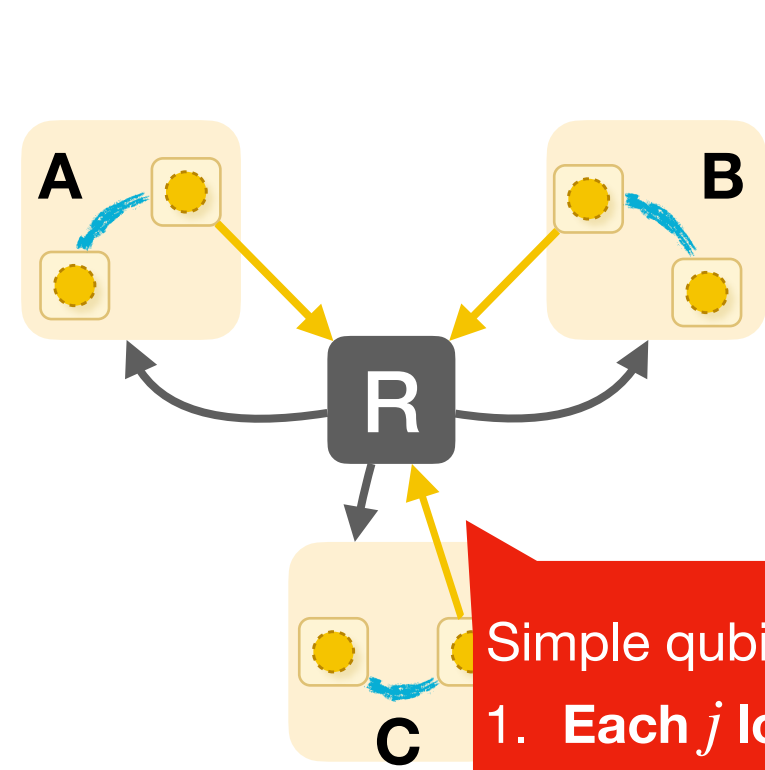
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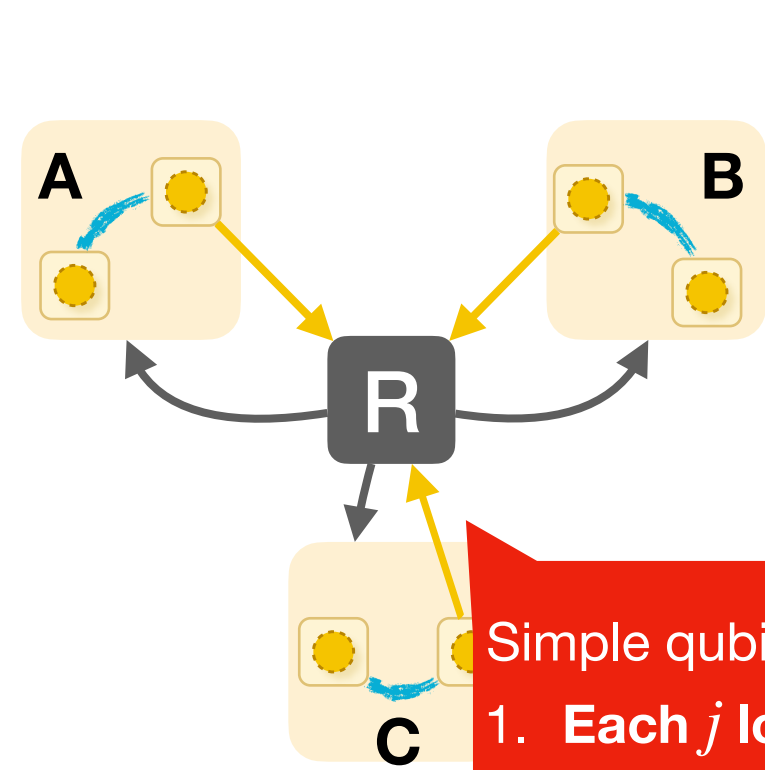
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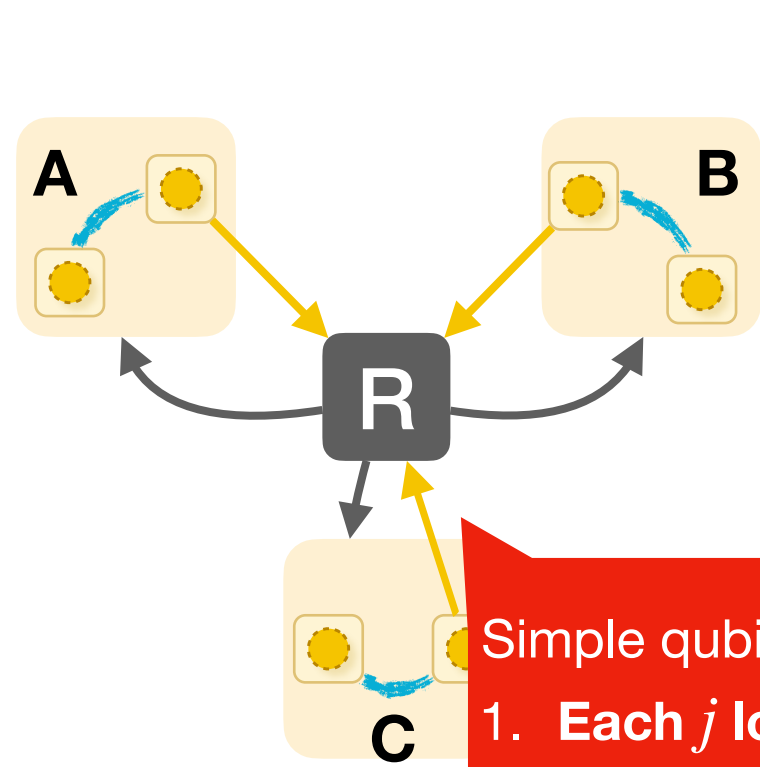
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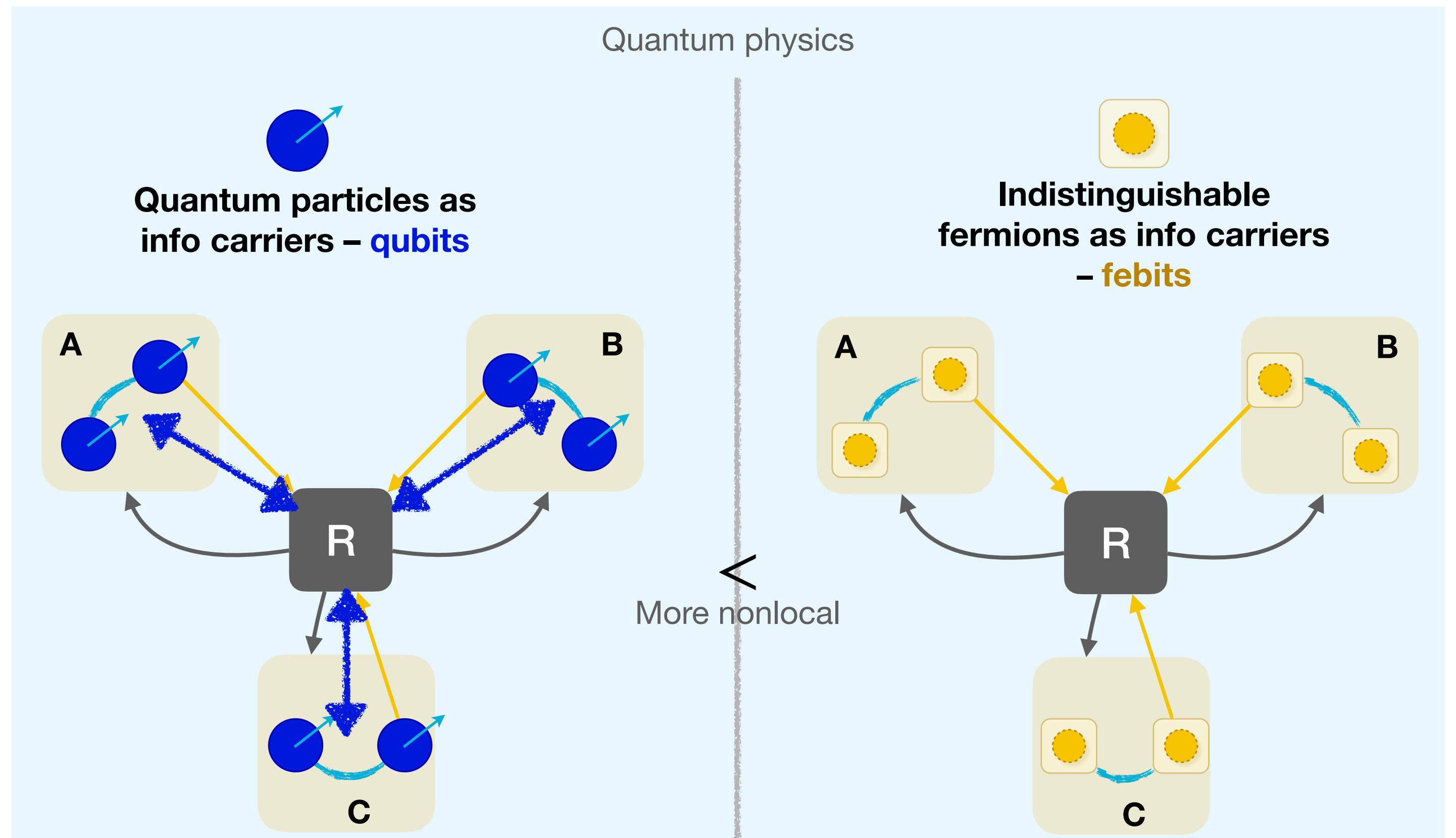
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Proof by contradiction:

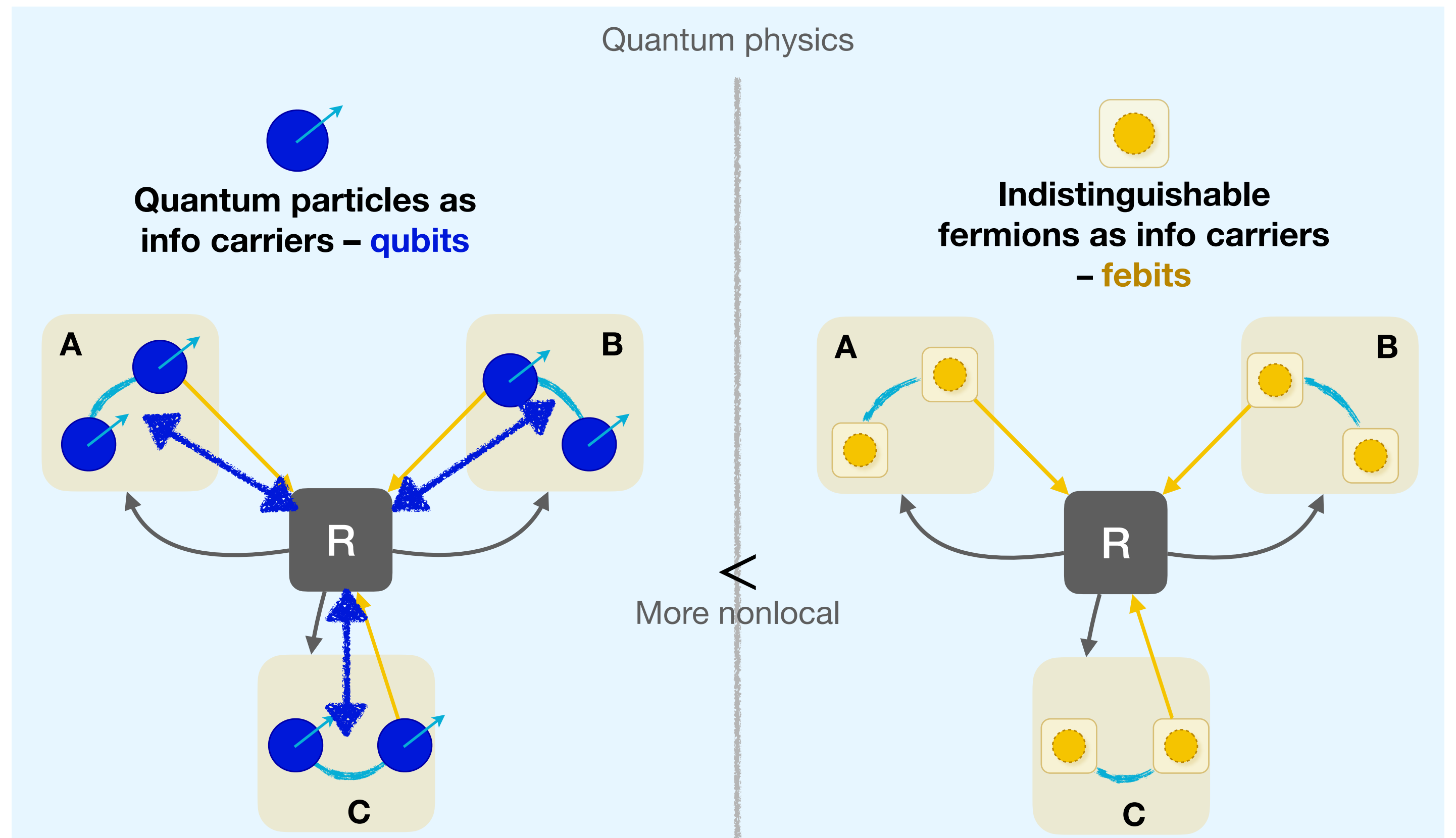
- suppose there is a general qubit strategy producing p_{febit}^l for all l
- For each l self-testing shows that this qubit strategy must be equivalent to the **simple qubit strategy**
- But the **simple qubit strategy** must be incompatible in the local phase $e^{i\phi_j}$ for different l !

Conclusion



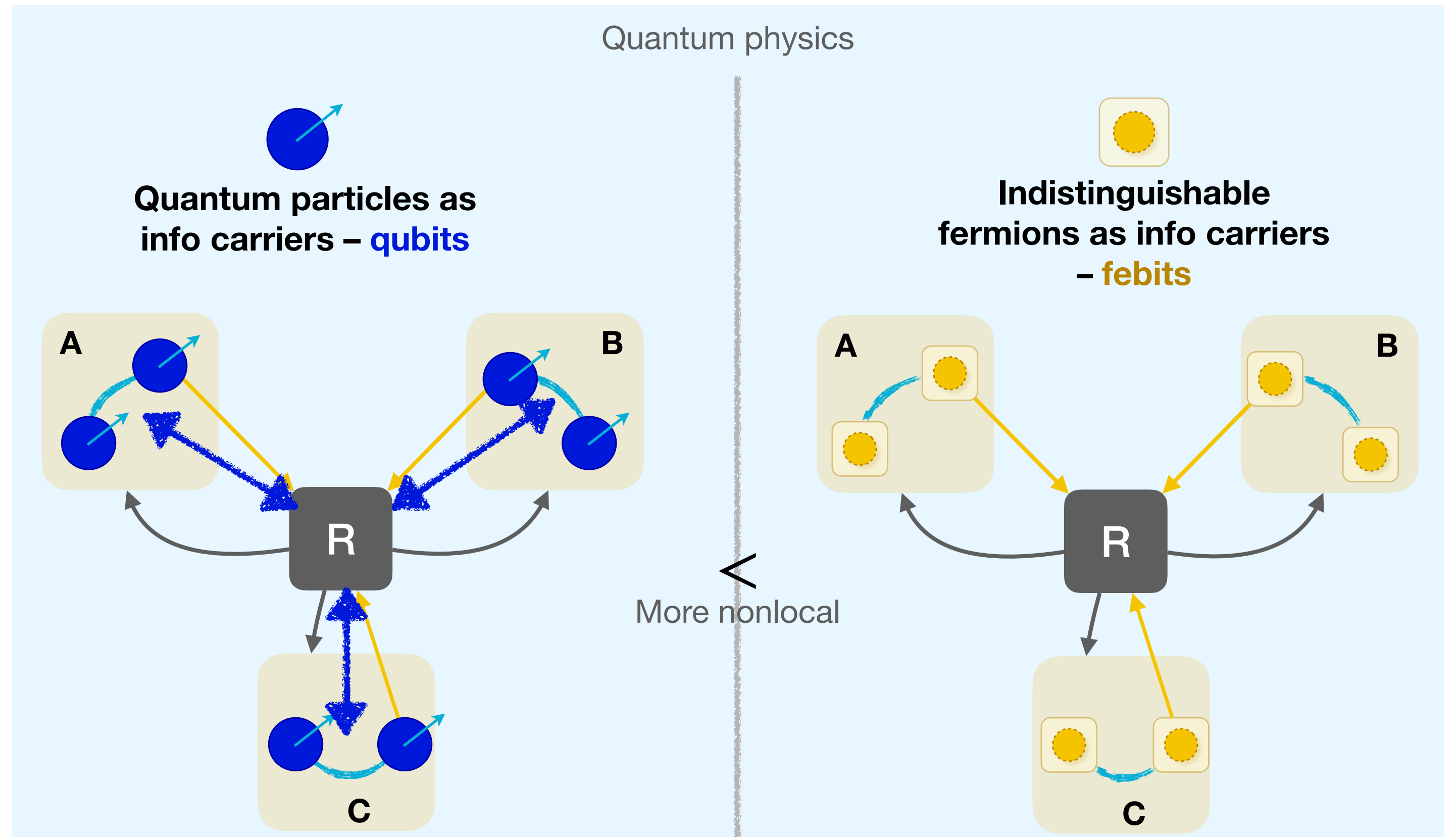
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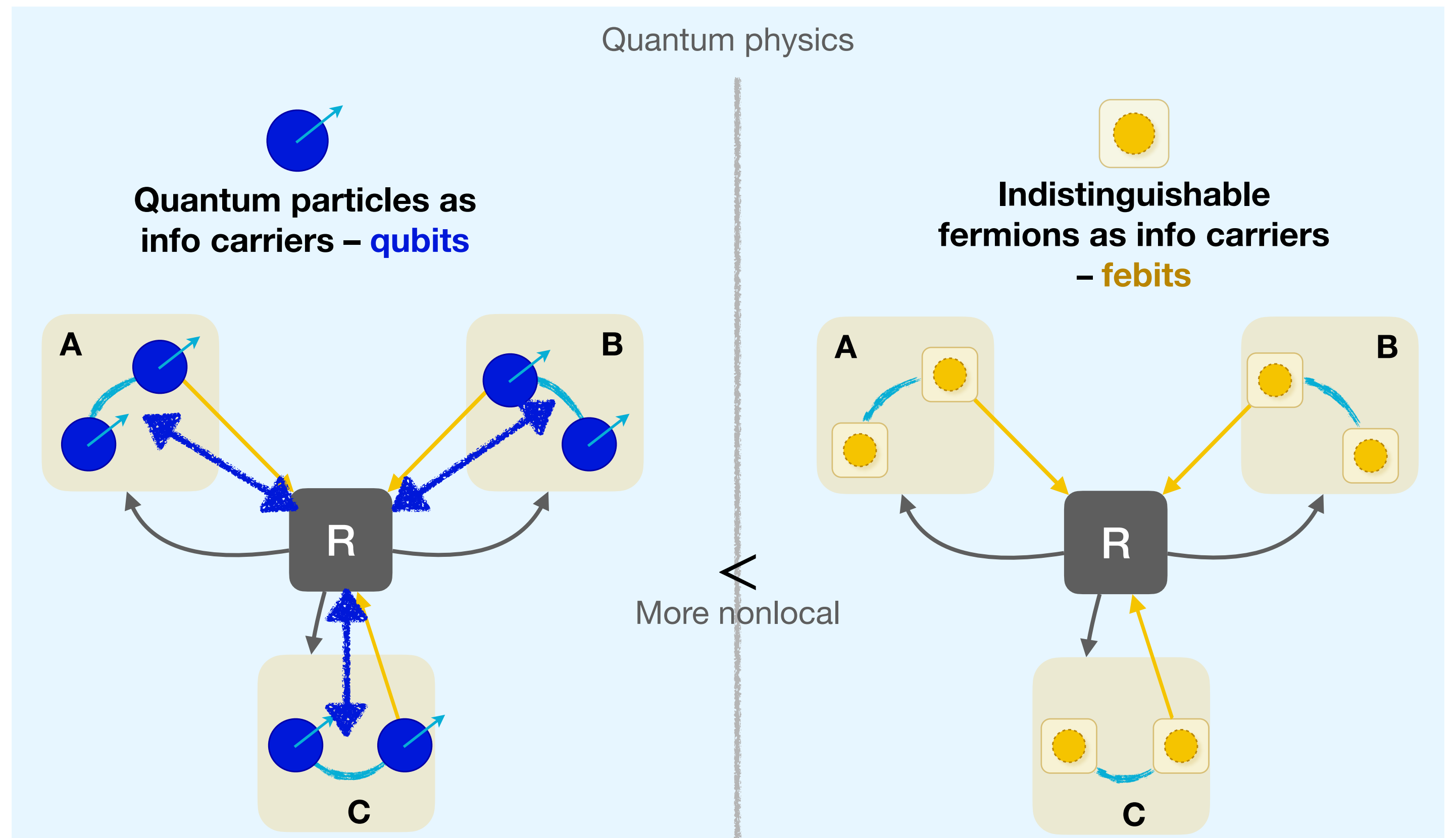
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- “=” if qubit have 2 rounds of communications instead of 1. A scenario with $O(N)$ -advantage?



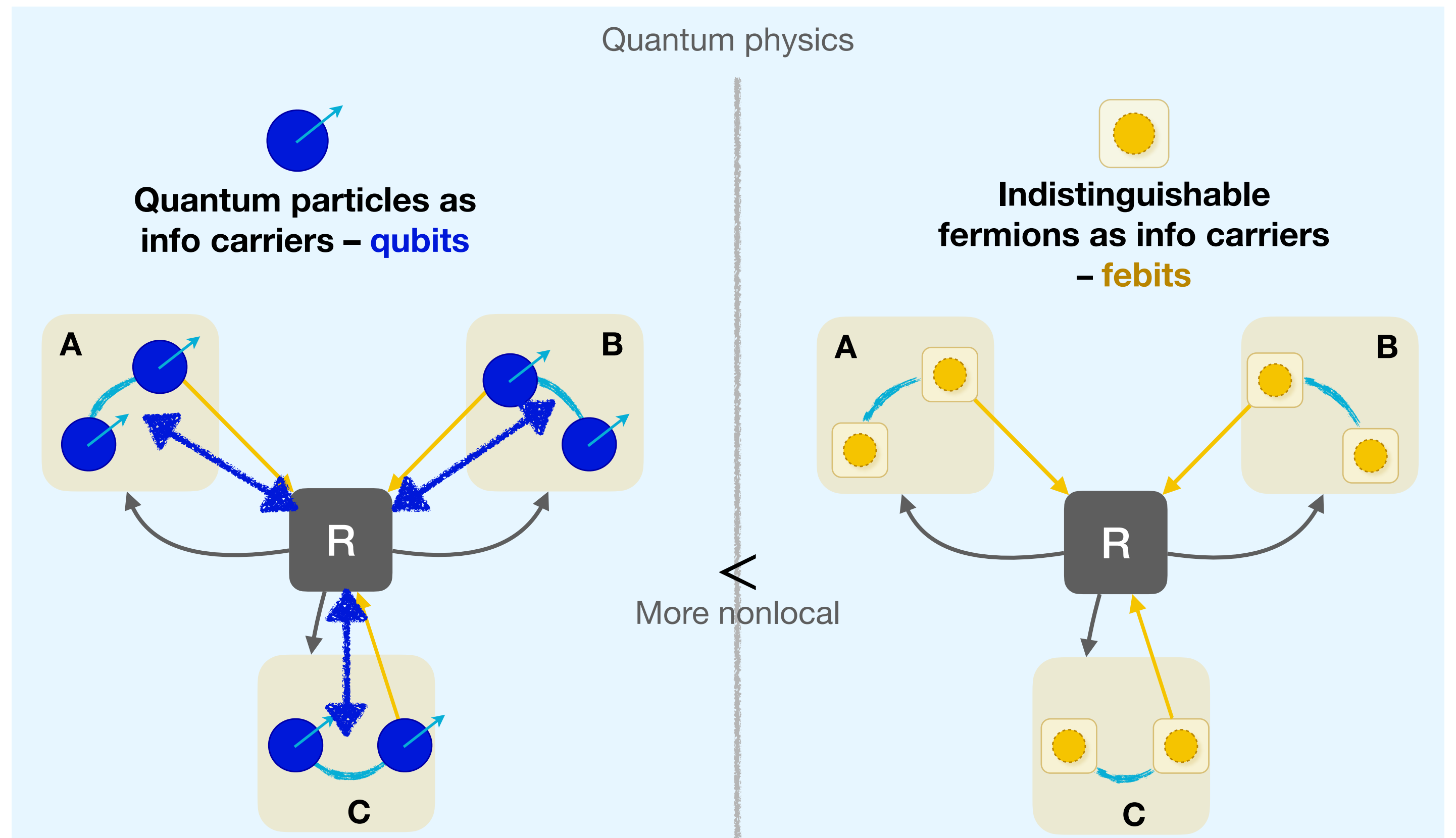
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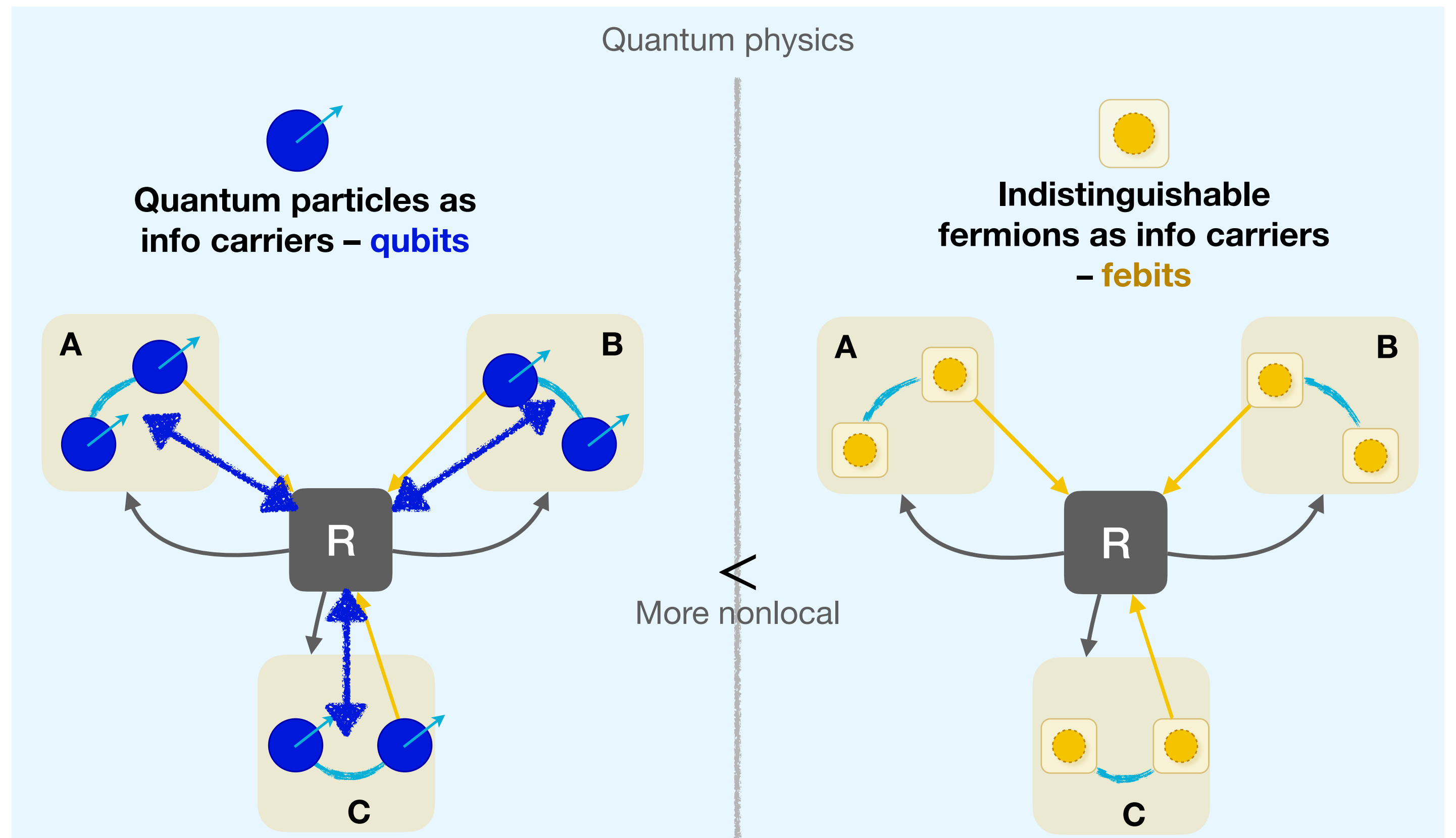
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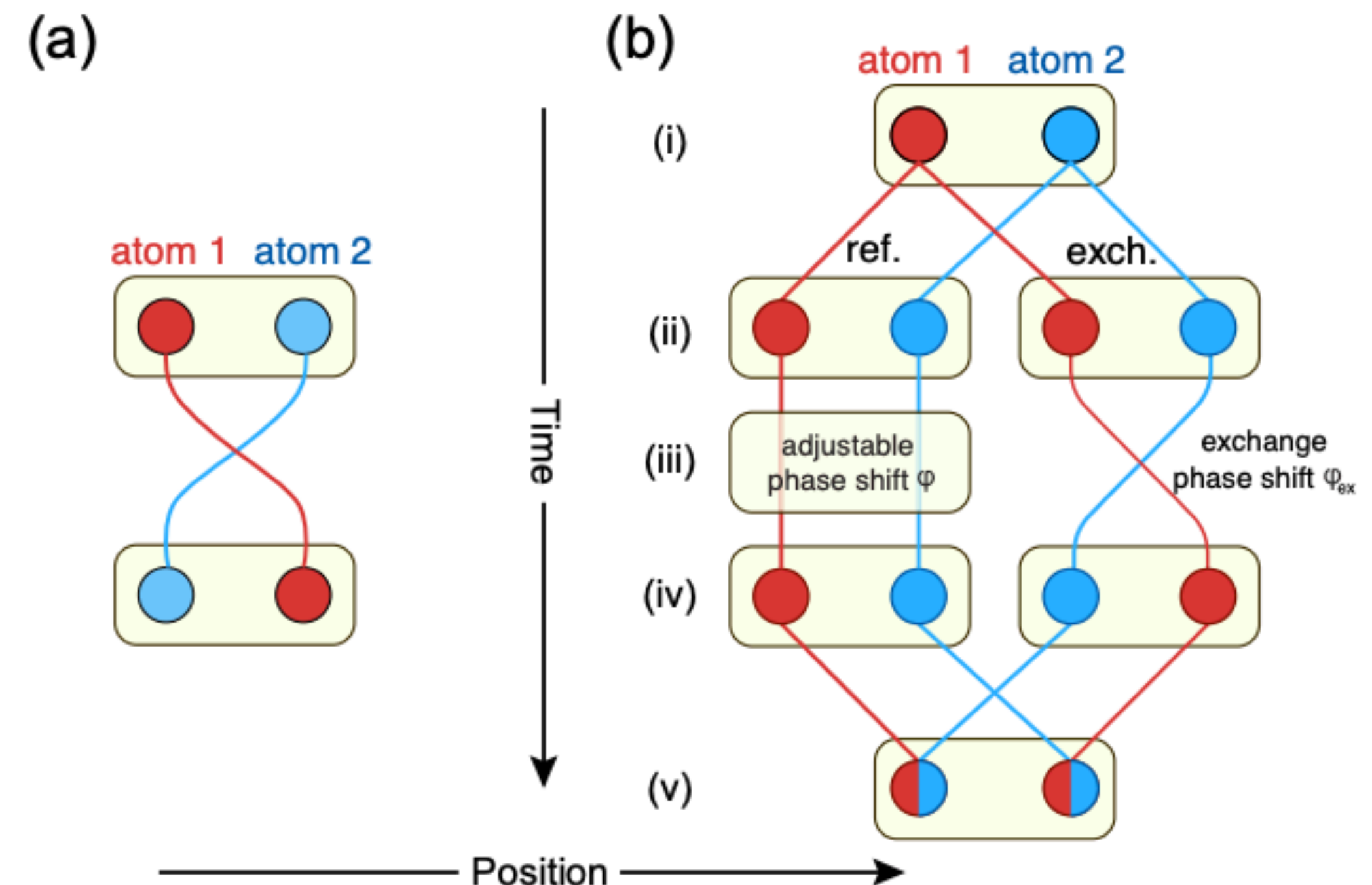
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- Separations for other particle statistics? *Anyons, paraparticles...*



Backup: Distant identical particles reveal exchange statistics?

- [Roos et al., *Revealing quantum statistics with a pair of distant atoms*, 2017]
- (a) Undetectable global phase
- (b) Reference + exchange gives relative phase
- $\phi = 0$ for bosons, $\phi = \pi$ for fermions
- Exchange statistics can be measured with distant massive particles, discussion on possible implementations, but device-dependent
- Our Gedankenexperiment takes this fact to the distributed network for a no-go theorem against qubits



Backup: mappings from febits to qubits are not local

- Jordan-Wigner transformation: mapping to encode febits with qubits
- $(f_1^\dagger)_1^n (f_2^\dagger)_2^n \cdots (f_m^\dagger)_m^n |\Omega\rangle := |n_1 n_2 \cdots n_m\rangle \mapsto |n_1 n_2 \cdots n_m\rangle$
- $1 - 2f_i^\dagger f_i := Z_i^{(f)} \mapsto Z_i$
- $f_i^\dagger + f_i := X_i^{(f)} \mapsto (\prod_{k=1}^{i-1} Z_k) X_i$
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