Fermions are fundamentally more nonlocal than Bosons

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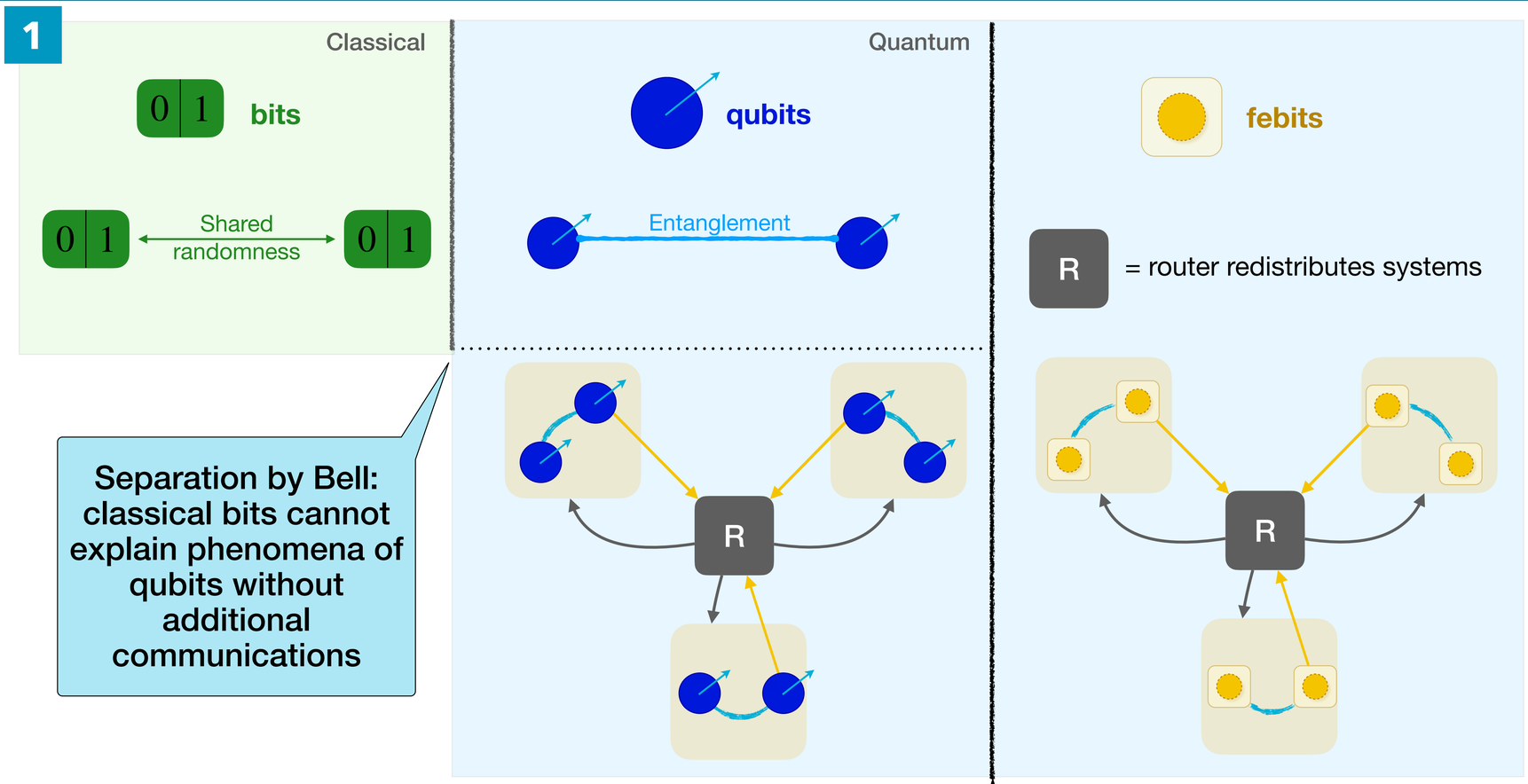


Separation by Bell: classical bits cannot explain phenomena of qubits without additional communications

Main result: qubits cannot explain phenomena of febits in distributed networks, unless with additional communication!

2

Qubit information theory

Encode information via

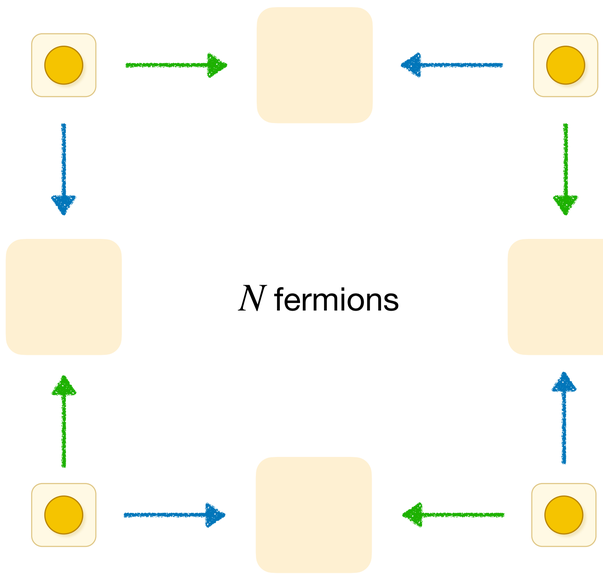
- Internal degrees of freedom: e.g., polarization of photons, spin of electrons, distinguishable particles
- Presence or absence of bosons
- Subsystems **commute**: $|11\rangle_{AB} = |11\rangle_{BA}$

Febit information theory

Encode information via (second quantization)

- Presence or absence of fermion
- = empty mode = $|0)$
- = one fermionic mode = $|1)$
- Fermionic modes **anticommute**: $|11)_{AB} = -|11)_{BA}$

3 Intuition for febit advantages

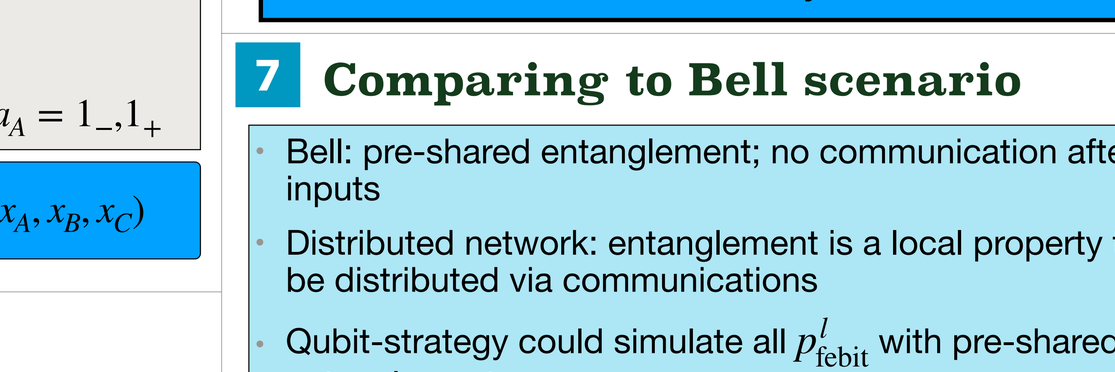


- Clockwise: $|\psi_c) = |1\cdots1)_{1,2\ldots,N}$
- Counterclockwise: $|\psi_a) = |1\cdots1)_{2,\cdots,N,1} = (-1)^{N-1}|\psi_c)$
- Superposition of $|\psi_c)$ and $|\psi_a)$ $\Longrightarrow$ **detectable relative phase** $(-1)^{N-1}$
- One step of communication already encodes the global information via this phase

But, if N is known, qubits can simulate febits by inserting this phase $(-1)^{N-1}$

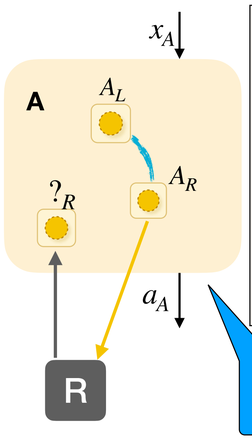
4 Hiding global configuration with a router

Depending on the router R, the network could have configuration l:



- Router is a passive relay, l is hidden from the players
- Qubit-based players cannot fix a phase in advance to reproduce all $(-1)^2, (-1)^1, (-1)^0$
- An example of distributed computing network in computer science

5 The Gedankenexperiment / fermionic strategy



1. **A** prepares $(|\square\bullet)_{A_LA_R} + |\bullet\square)_{A_LA_R})/\sqrt{2} = (|01)_{A_LA_R} + |10)_{A_LA_R})/\sqrt{2}$; sends A_R to **R**
2. **R** secretly chooses a configuration l and redistributes the $?_R$ systems; sends input $x_i \in \{0,1\}$ to every player i
3. **A** locally counts number of fermions $n_A \in \{0,1,2\}$:
   - If $n_A = 0$, output $a_A = 0$; $n_A = 2$, output $a_A = 2$
   - If $n_A = 1$, perform two orthogonal projective measurements $\Pi_-^{x_A}, \Pi_+^{x_A}$ for $a_A = 1_-, 1_+$

For each hidden configuration l, this gives a correlation $p^l_{\text{febit}}(a_A, a_B, a_C \mid x_A, x_B, x_C)$

6 Main theorem (informal)

Under independent local preparation, hidden l, and the same single communication round, no qubit-based strategy can reproduce p^l_{febit} for all configurations l simultaneously.

7 Comparing to Bell scenario

- Bell: pre-shared entanglement; no communication after inputs
- Distributed network: entanglement is a local property to be distributed via communications
- Qubit-strategy could simulate all p^l_{febit} with pre-shared entanglement

8 Proof idea with self-testing

For a fixed configuration: "naive qubit simulation for l"

- Each player j prepares local states $(|01\rangle_{j_Lj_R} + e^{i\pi\phi_j}|10\rangle_{j_Lj_R})/\sqrt{2}$
- Suitable local phase $\{\phi_j\}$ to recover the relative phase "$(-1)^{N-1}$"
- This can reproduce $p^l_{\text{febit}}(a_A, a_B, a_C \mid x_A, x_B, x_C)$

Self-testing argument (main technicality)

- Theorem: *Any qubit strategy* of arbitrary dimensions that reproduce p^l_{febit} must be essentially equivalent to this "naive qubit simulation for l"
- Ingredients: partial self-testing for token count games; GHZ self-testing

But no single choice of $\{\phi_j\}$ can reproduce all p^l_{febit} simultaneously

A proof by contradiction leads to the main theorem

- **Take-home message:** fermions/febits have more information-processing potential than qubits in quantum theory.
- **Open**: Robustness; scalable fermionic advantages; experimental implementations; anyons and paraparticles?

arXiv:2606.12363